Beyond Linear Regression

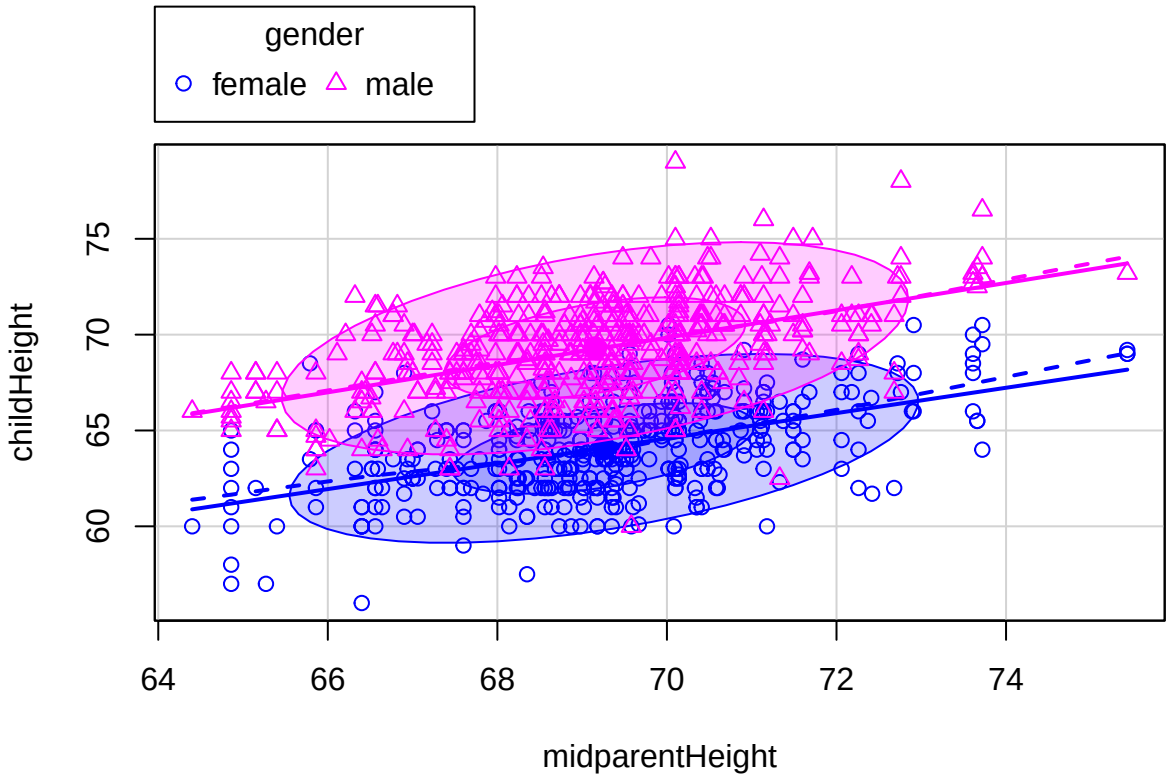
Load the Galton height data from the `HistData` library.

```
data(GaltonFamilies)
str(GaltonFamilies)
```

```
## 'data.frame':    934 obs. of  8 variables:
##  $ family         : Factor w/ 205 levels "001","002","003",..: 1 1 1 1 2 2 2 2 3 3 ...
##  $ father         : num  78.5 78.5 78.5 78.5 75.5 75.5 75.5 75.5 75 75 ...
##  $ mother         : num  67 67 67 67 66.5 66.5 66.5 66.5 64 64 ...
##  $ midparentHeight: num  75.4 75.4 75.4 75.4 73.7 ...
##  $ children       : int  4 4 4 4 4 4 4 4 2 2 ...
##  $ childNum       : int  1 2 3 4 1 2 3 4 1 2 ...
##  $ gender         : Factor w/ 2 levels "female","male": 2 1 1 1 2 2 1 1 2 1 ...
##  $ childHeight    : num  73.2 69.2 69 69 73.5 72.5 65.5 65.5 71 68 ...
```

Reproduce Fig 2 in Hanley (2004).

```
scatterplot(childHeight ~ midparentHeight | gender, data=GaltonFamilies,
    ellipse=TRUE, levels=0.68, legend.coords=list(x=64, y=78))
```



Data exploration

```
nrow(GaltonFamilies)
```

```
## [1] 934
```

```
length(unique(GaltonFamilies$family))
```

```
## [1] 205
```

```
summary(GaltonFamilies$children)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   1.000   4.000   6.000   6.171   8.000  15.000
```

```
table(GaltonFamilies$gender)
```

```
##
## female   male
##    453    481
```

```
summary(GaltonFamilies$childHeight)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   56.00   64.00   66.50   66.75   69.70   79.00
```

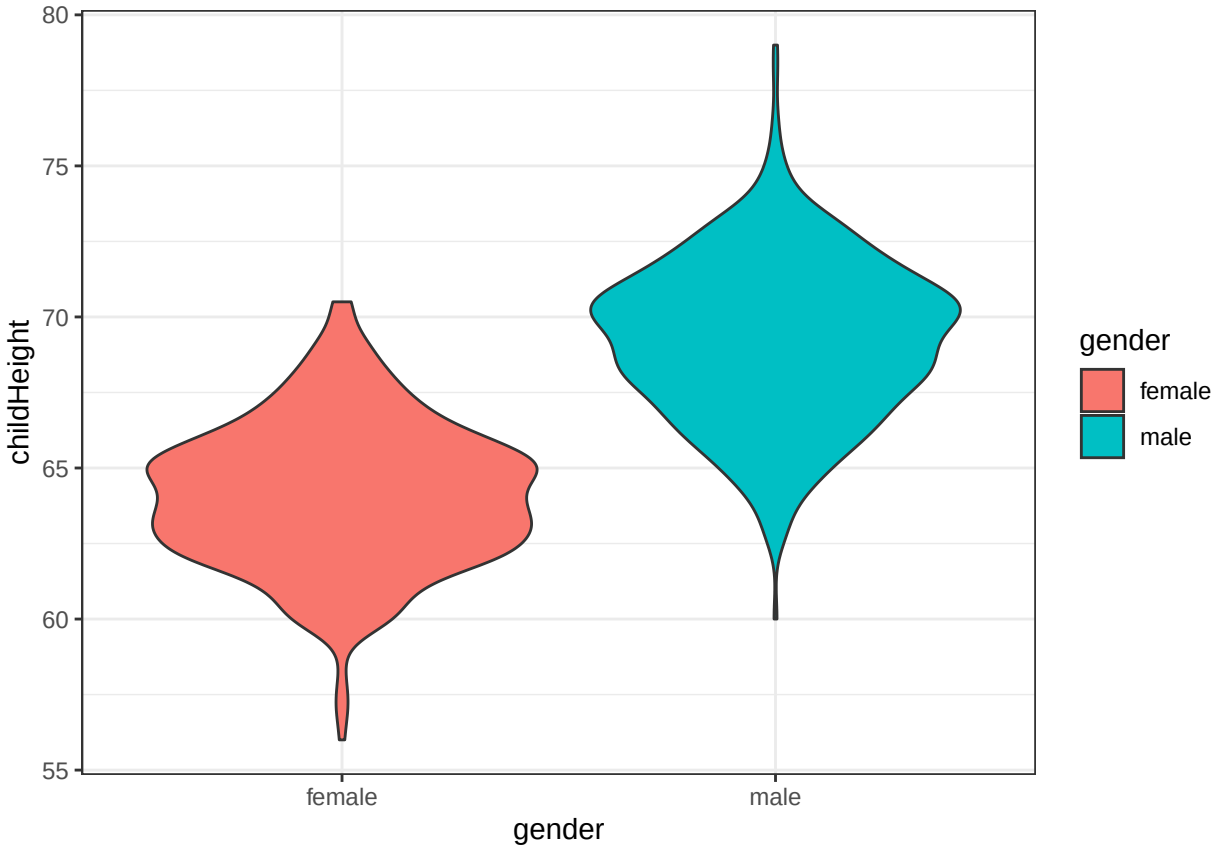
```
sd(GaltonFamilies$father)
```

```
## [1] 2.476479
```

```
sd(GaltonFamilies$mother)
```

```
## [1] 2.290886
```

```
ggplot(GaltonFamilies, aes(x=gender, y=childHeight, fill=gender)) + geom_violin() + theme_bw()
```

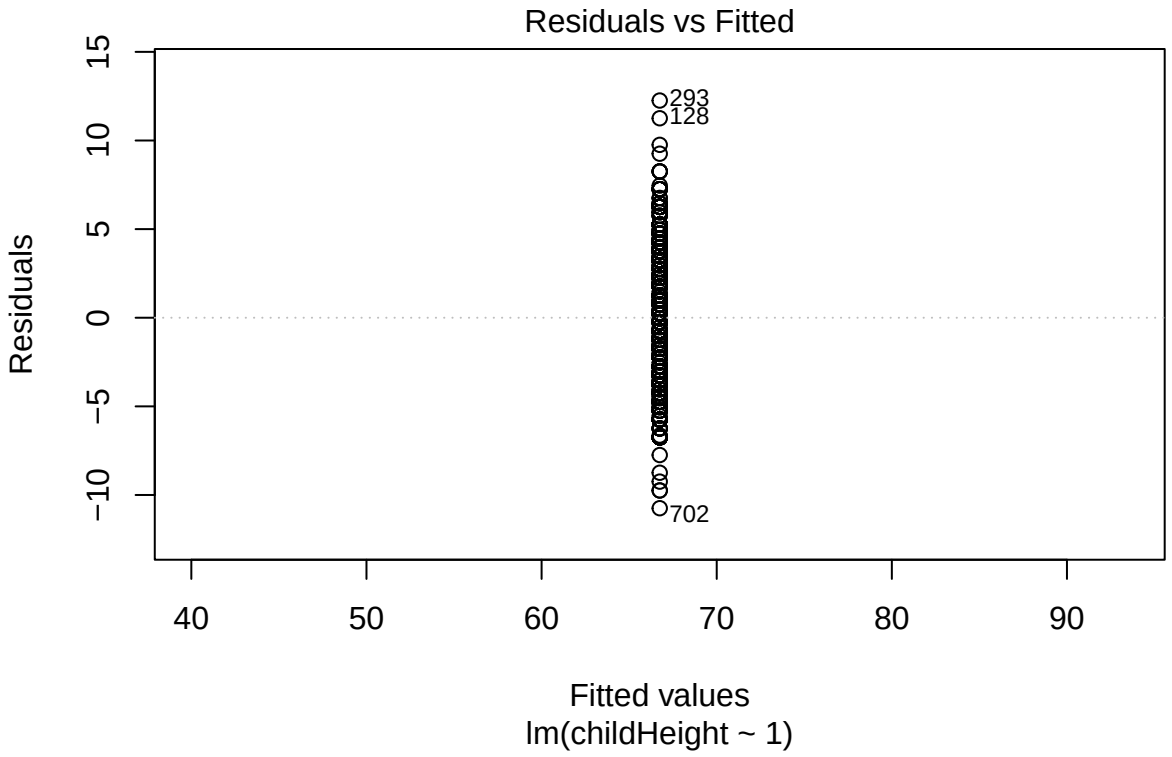


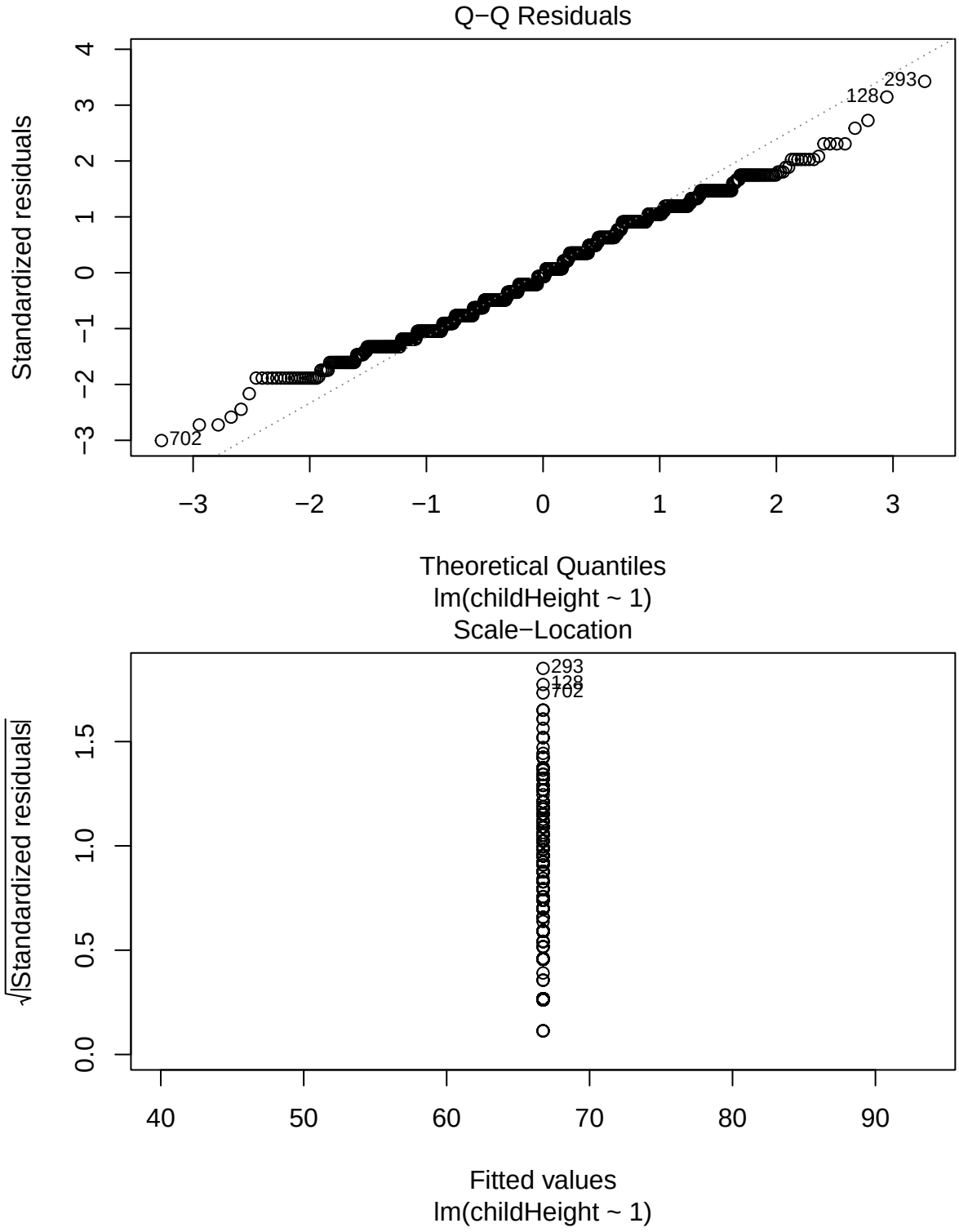
What's the simplest possible model of this data you could imagine? Probably one that just computes the mean height.

```
m0 <- lm(childHeight ~ 1, data=GaltonFamilies)
summary(m0)
```

```
##
## Call:
## lm(formula = childHeight ~ 1, data = GaltonFamilies)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10.7459  -2.7459  -0.2459   2.9541  12.2541
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  66.7459     0.1171   569.9   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.579 on 933 degrees of freedom
```

```
plot(m0)
```



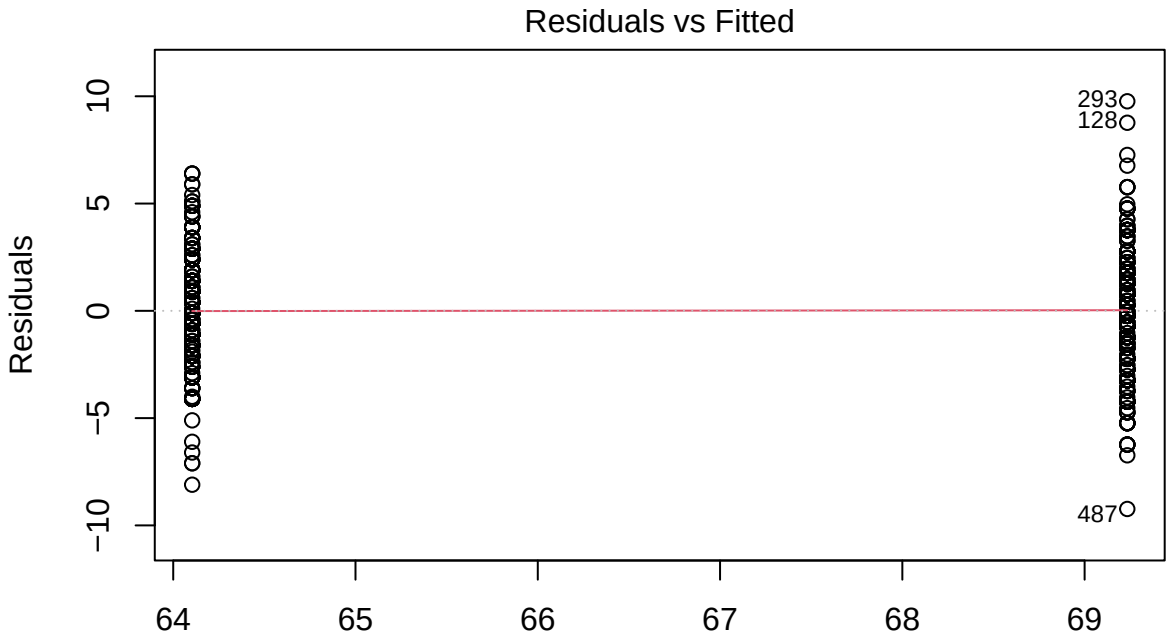

Q−Q Residuals
Standardized residuals
Theoretical Quantiles
lm(childHeight ~ 1)
Scale−Location
√|Standardized residuals|
Fitted values
lm(childHeight ~ 1)
293
128
702

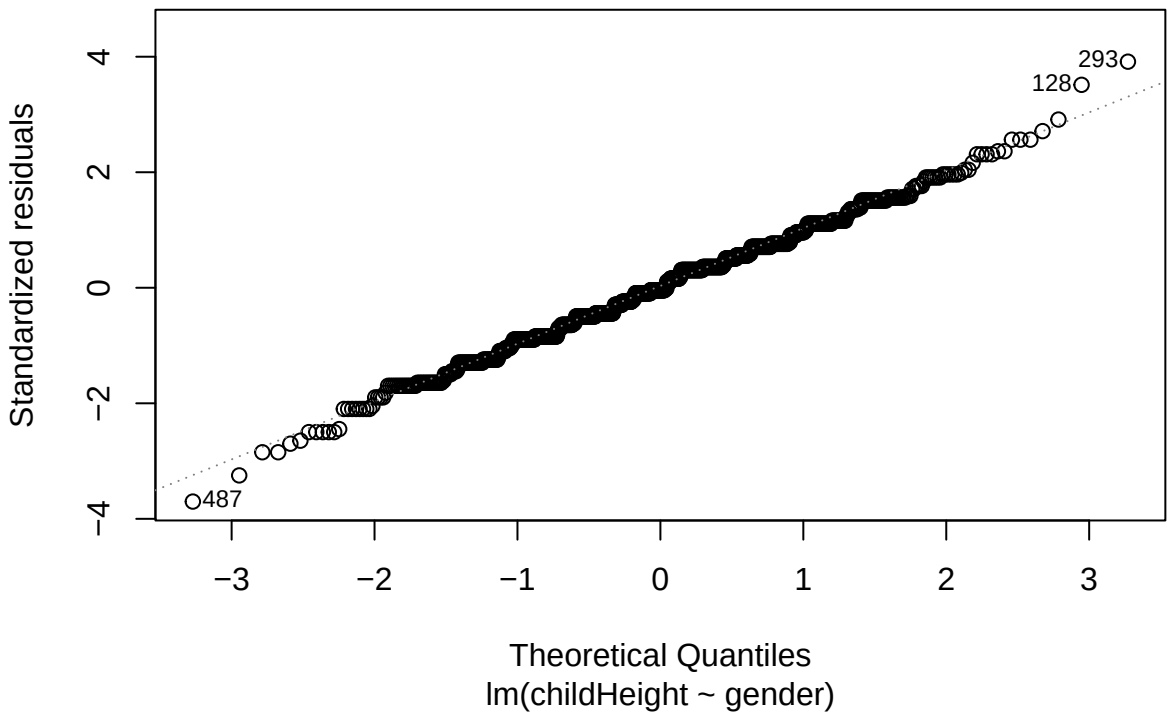
Let's make it more realistic. Fit a simple regression model predicting children's height from gender.

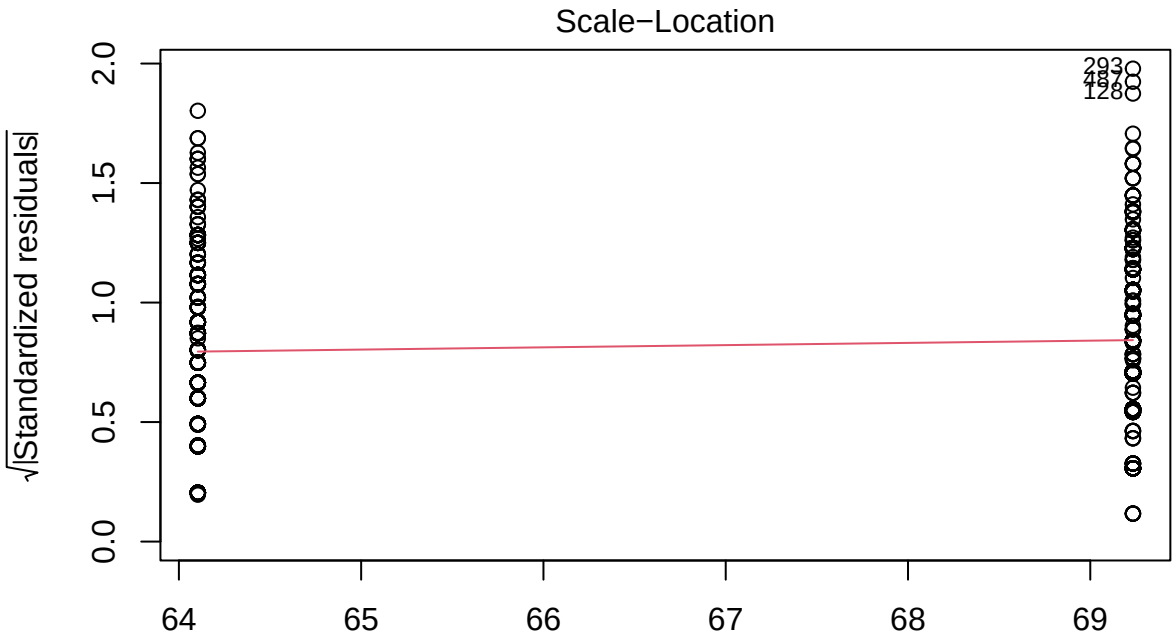
```
m1 = lm(childHeight ~ gender, data=GaltonFamilies)
summary(m1)
```

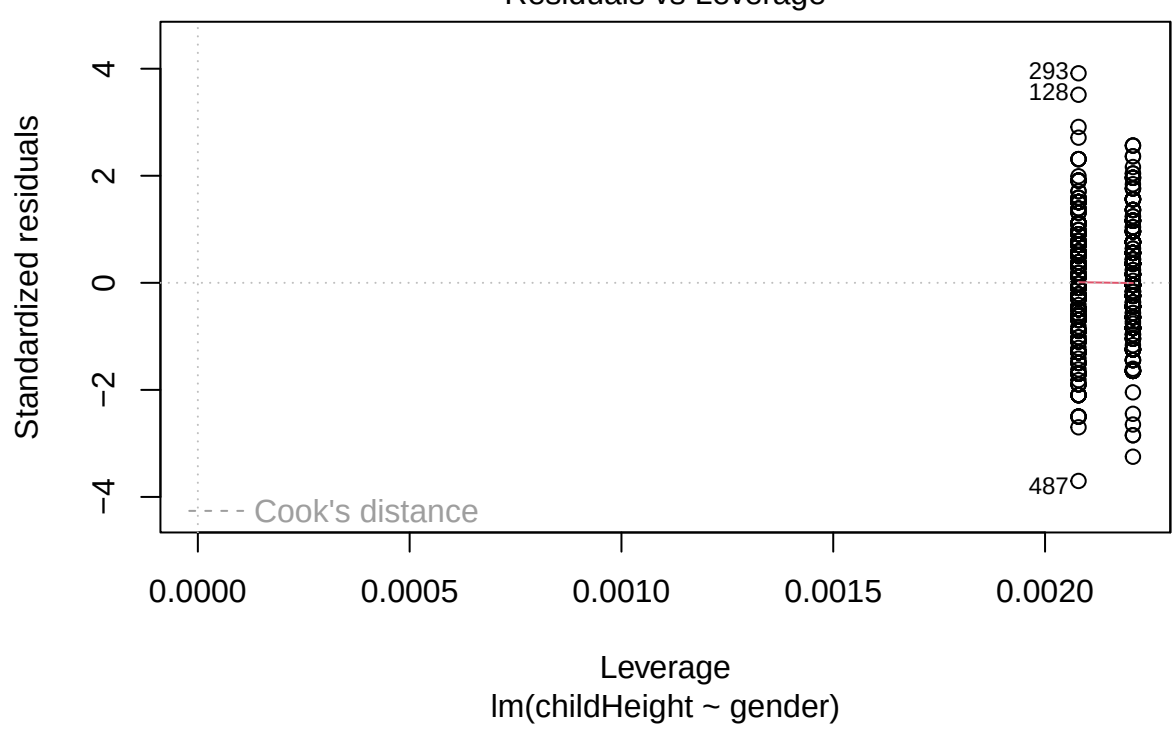
```
##
## Call:
## lm(formula = childHeight ~ gender, data = GaltonFamilies)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -9.234 -1.604 -0.104  1.766  9.766
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  64.1040     0.1173  546.32   <2e-16 ***
## gendermale    5.1301     0.1635   31.38   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.497 on 932 degrees of freedom
## Multiple R-squared:  0.5137, Adjusted R-squared:  0.5132
## F-statistic: 984.4 on 1 and 932 DF,  p-value: < 2.2e-16
```

```
plot(m1)
```


Residuals vs Fitted
Residuals
10
5
0
−5
−10
293
128
487
64
65
66
67
68
69
Fitted values
lm(childHeight ~ gender)

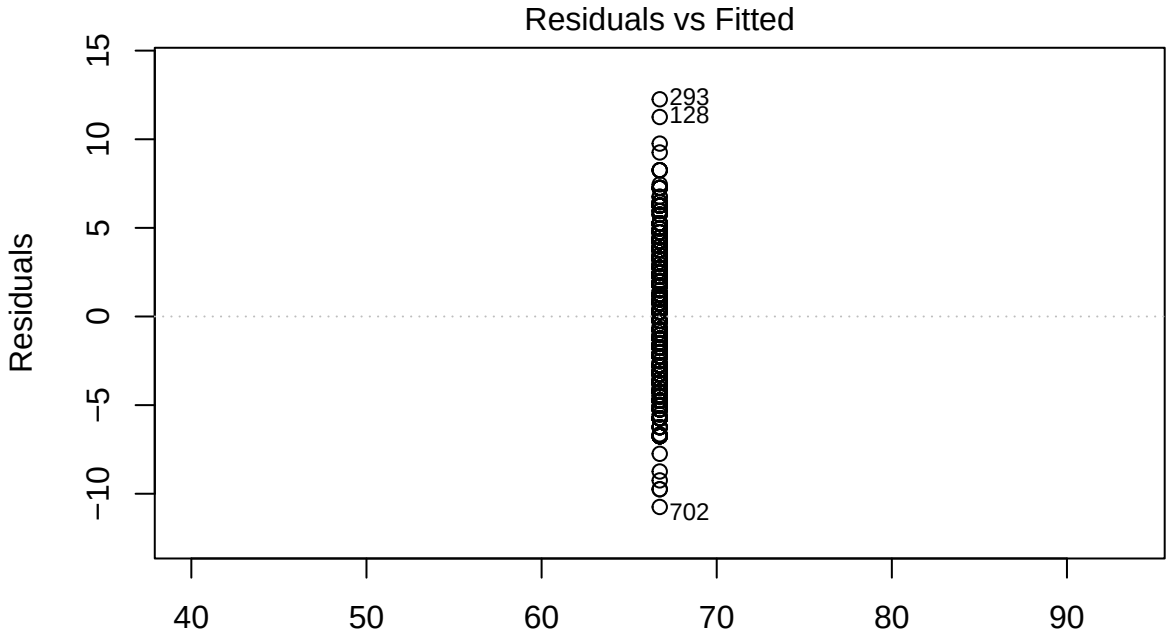

Q−Q Residuals
Standardized residuals
4
2
0
−2
−4
293
128
487
−3
−2
−1
0
1
2
3
Theoretical Quantiles
lm(childHeight ~ gender)

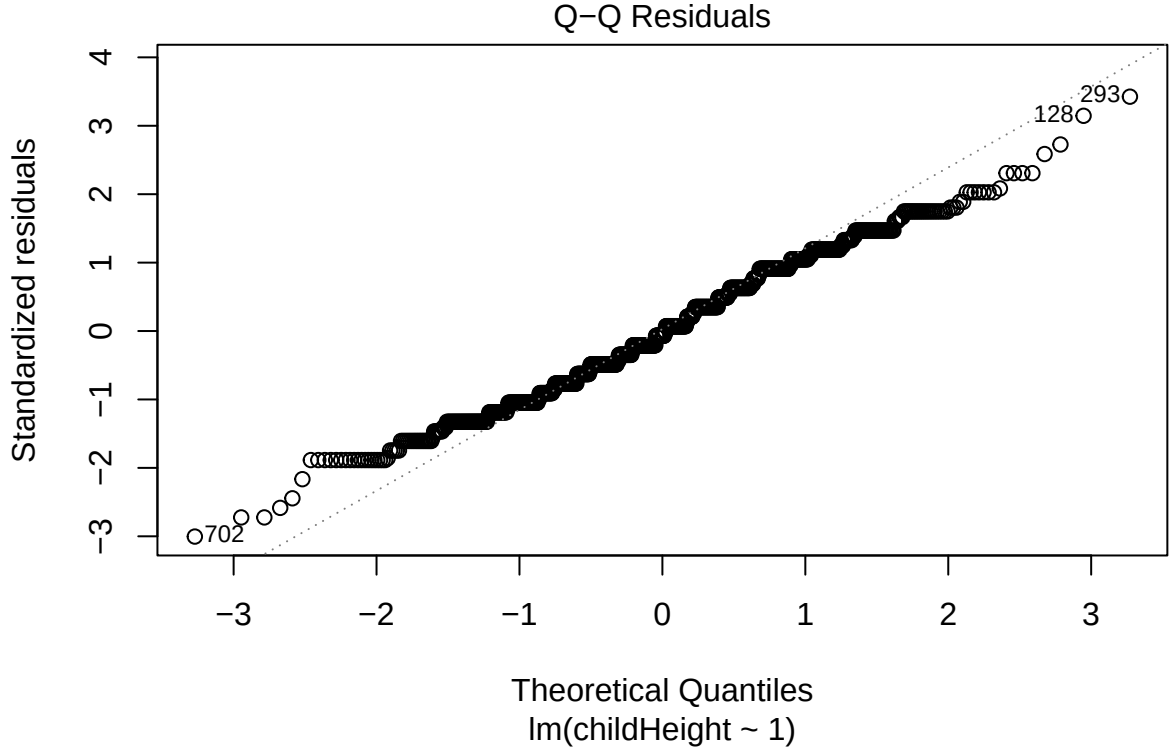


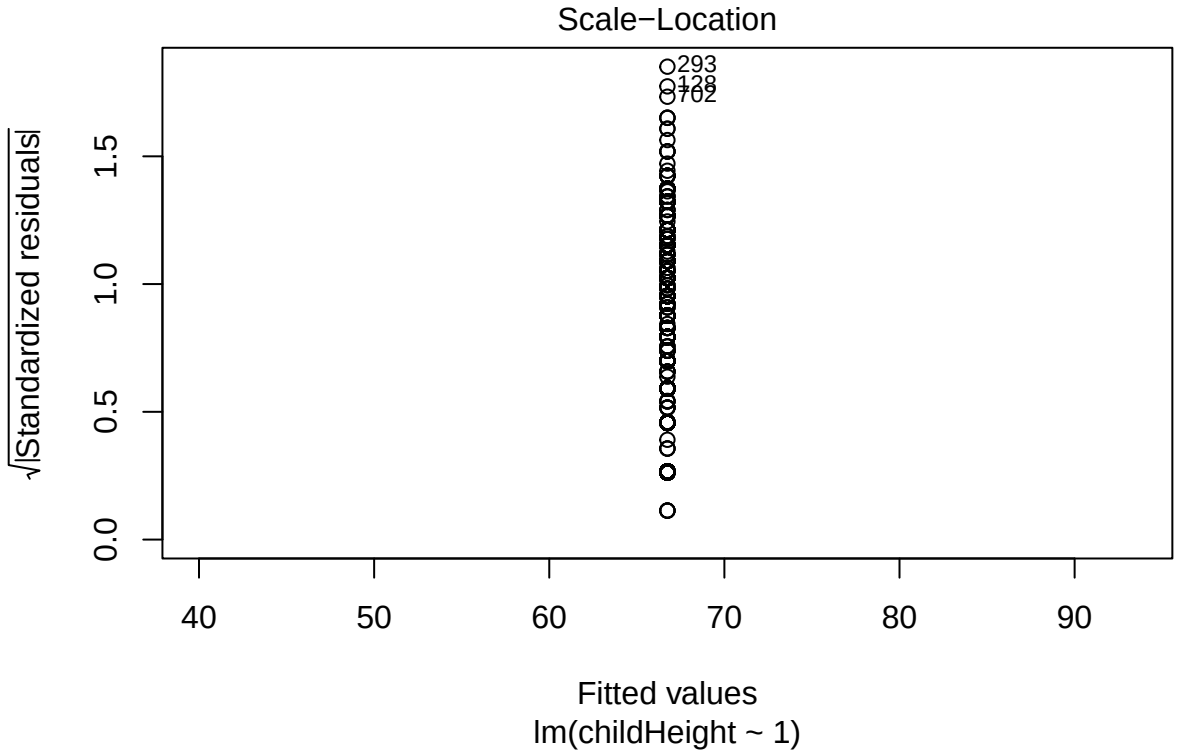


Look at some model diagnostics to confirm that this is an appropriate model. Since this model only produces two different predictions (one for males and one for females), that isn't very helpful here.

```
plot(m0)
```

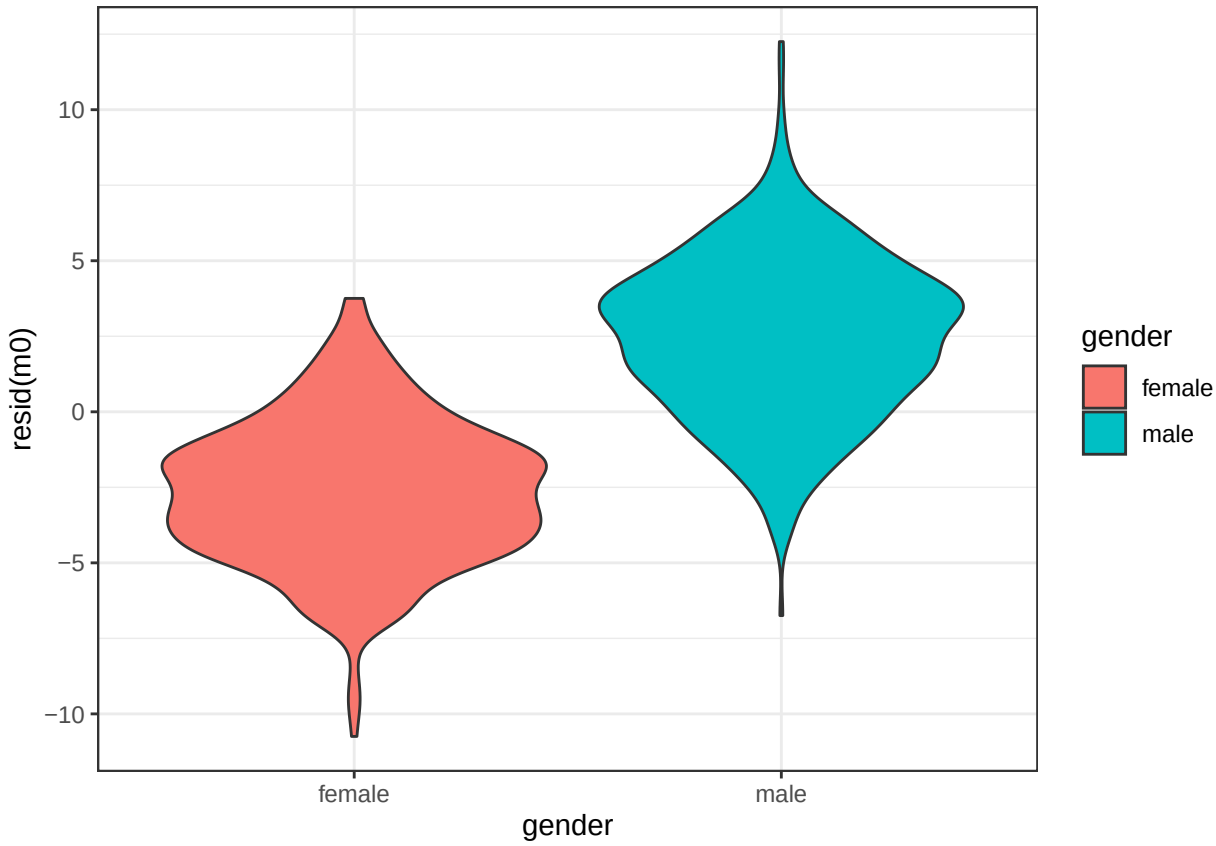

Residuals vs Fitted
Residuals
15
10
5
0
−5
−10
293
128
702
40
50
60
70
80
90
Fitted values
lm(childHeight ~ 1)


Q−Q Residuals
Standardized residuals
4
3
2
1
0
−1
−2
−3
293
128
702
−3
−2
−1
0
1
2
3
Theoretical Quantiles
lm(childHeight ~ 1)



A boxplot or violin plot can help to summarise the distribution of residuals by group. Since the model simply estimates the mean heights of males and females a violin plot of the residuals should look very similar to the violin plot of heights above, but with the means of both groups aligned at 0.

```
ggplot(GaltonFamilies, aes(x=gender, y=resid(m0), fill=gender)) +
   geom_violin() +
   theme_bw()
```



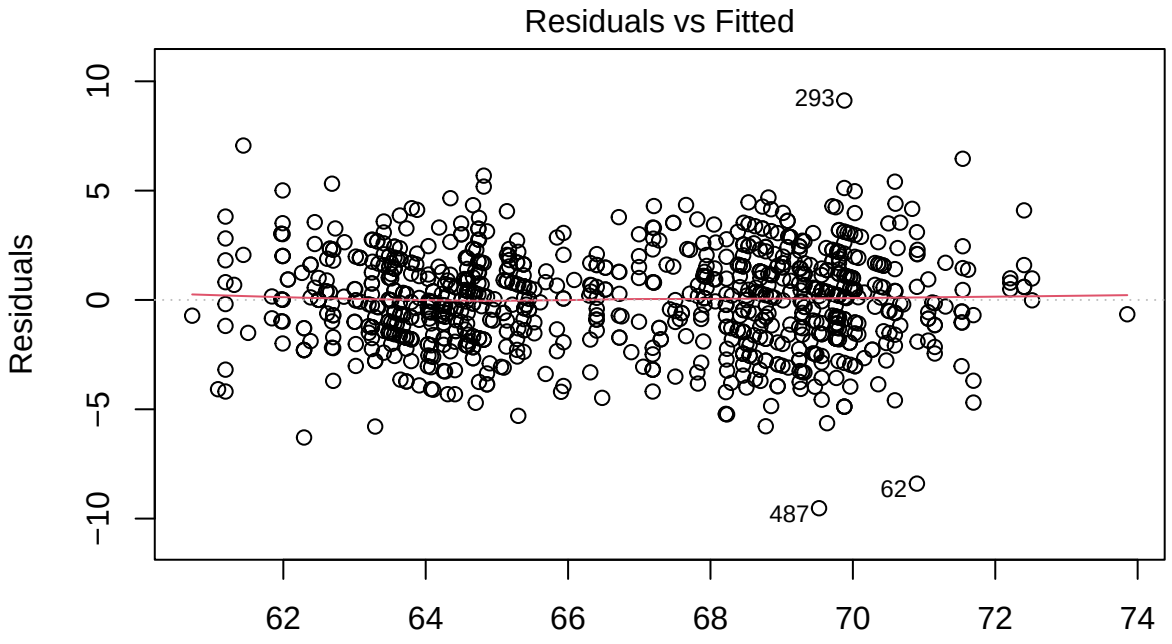
Fit a multiple regression model predicting children's height from father's height, mother's height, and gender

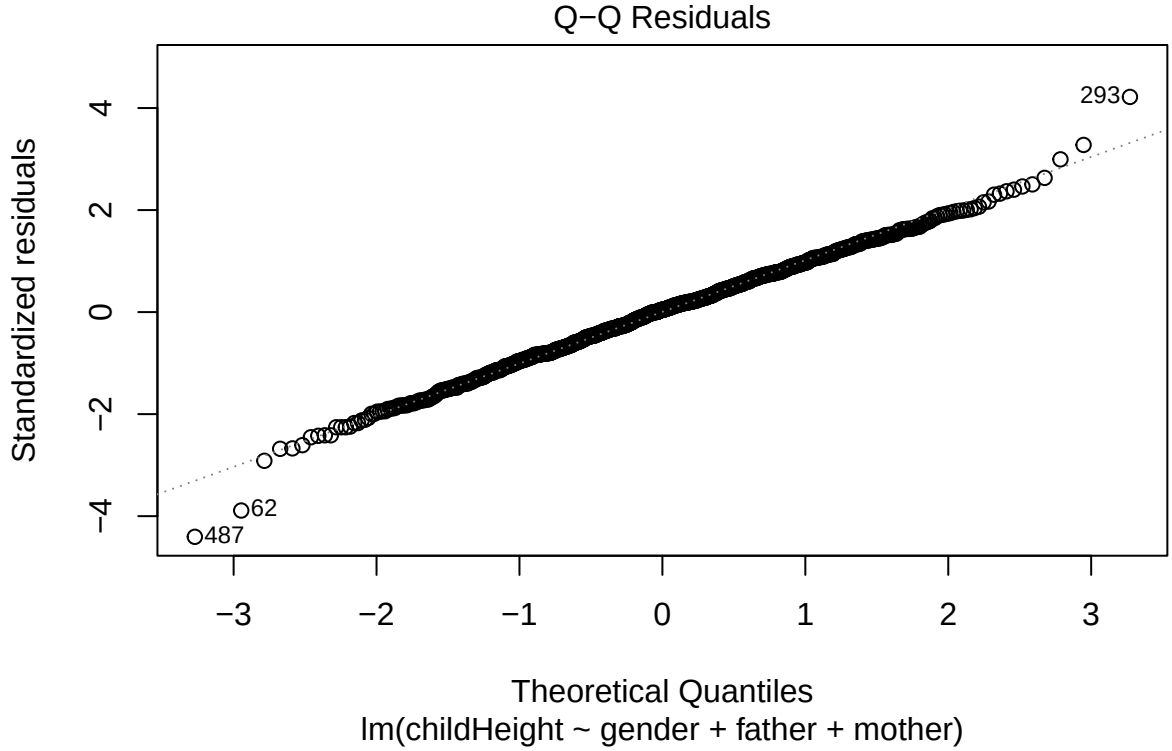
```
m2 = lm(childHeight ~ gender + father + mother, data=GaltonFamilies)
summary(m2)
```

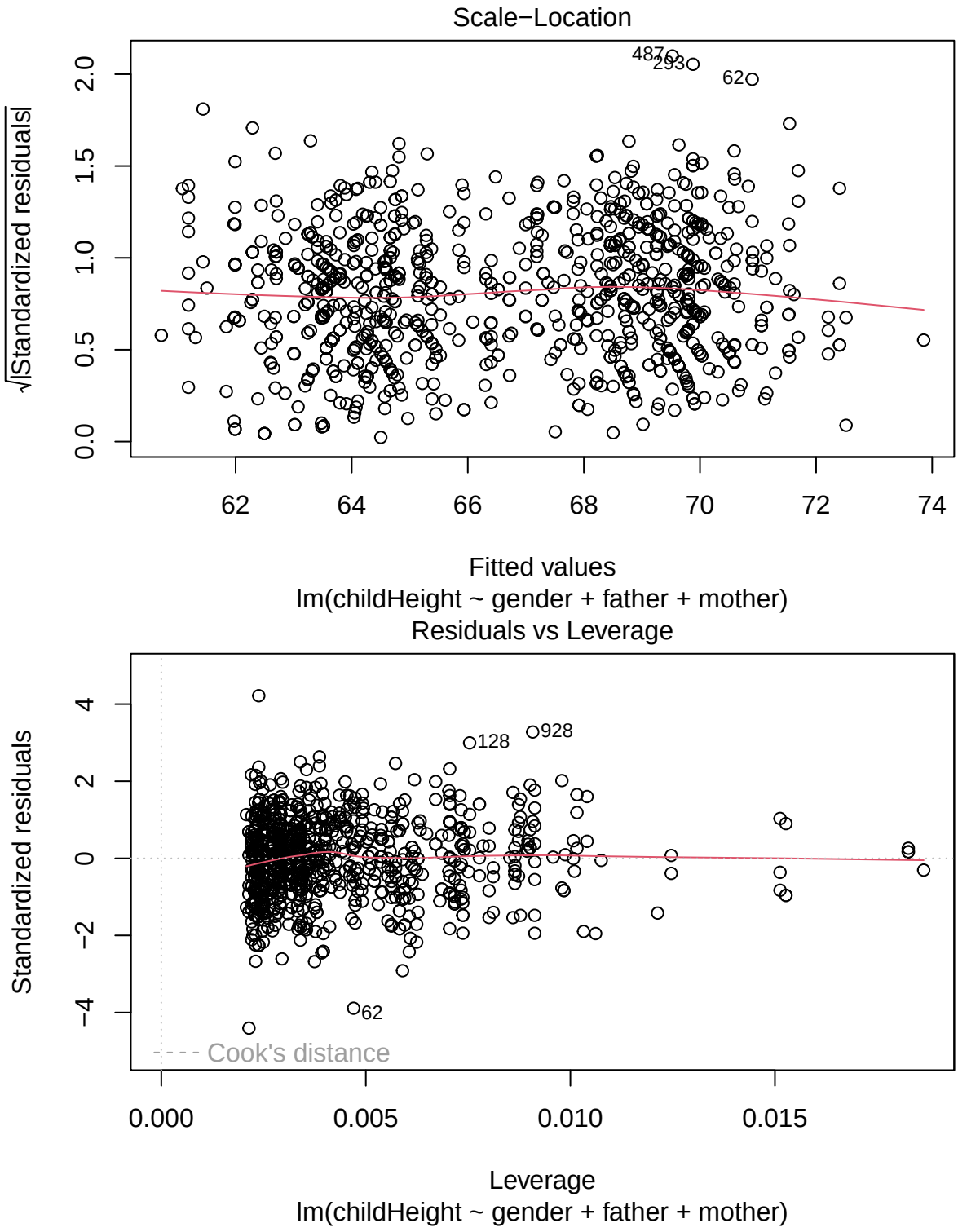
```
##
## Call:
## lm(formula = childHeight ~ gender + father + mother, data = GaltonFamilies)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.5247 -1.4653  0.0943  1.4860  9.1201
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 16.52124    2.72720   6.058    2e-09 ***
## gendermale   5.21499    0.14181  36.775   <2e-16 ***
## father       0.39284    0.02868  13.699   <2e-16 ***
## mother       0.31761    0.03100  10.245   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.165 on 930 degrees of freedom
## Multiple R-squared:  0.6354, Adjusted R-squared:  0.6342
```

```
## F-statistic: 540.3 on 3 and 930 DF,  p-value: < 2.2e-16
```

```
plot(m2)
```







Change reference level for gender variable.

```
levels(GaltonFamilies$gender)
```

```
## [1] "female" "male"
```

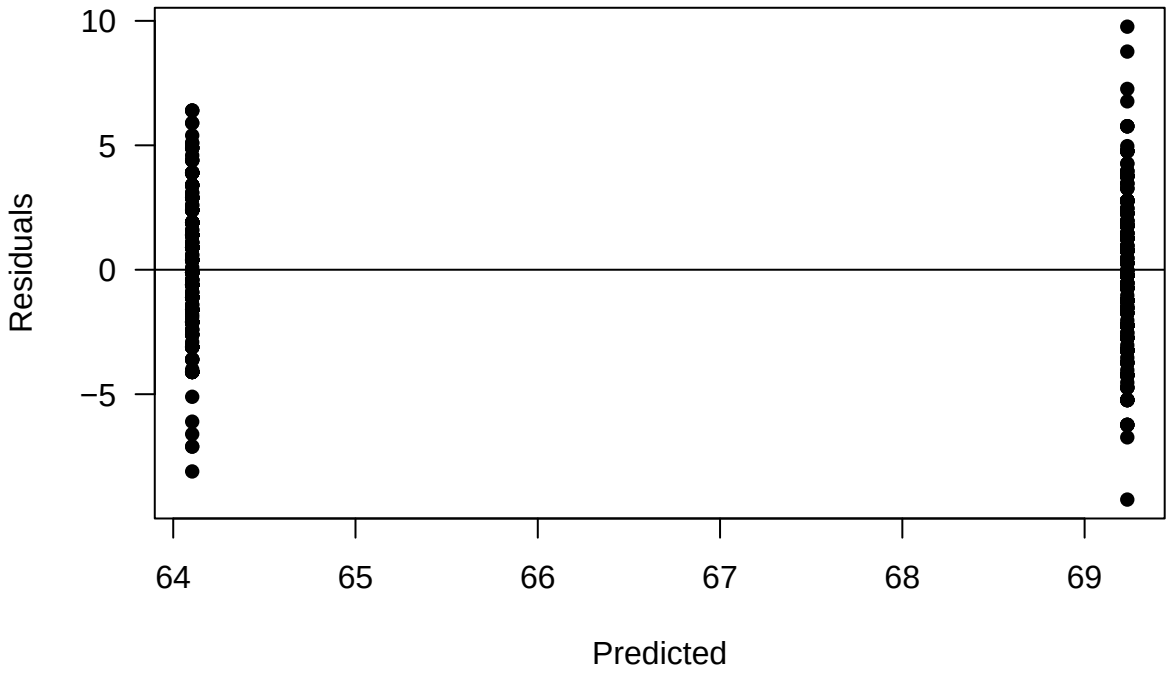
```r
GaltonFamilies$gender = factor(GaltonFamilies$gender, levels = c("male", "female"))

# The regression
m3 = lm(childHeight ~ gender + father + mother, data=GaltonFamilies)
summary(m3)
```

```
##
## Call:
## lm(formula = childHeight ~ gender + father + mother, data = GaltonFamilies)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.5247 -1.4653  0.0943  1.4860  9.1201
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  21.73623    2.72223   7.985 4.14e-15 ***
## genderfemale -5.21499    0.14181 -36.775  < 2e-16 ***
## father        0.39284    0.02868  13.699  < 2e-16 ***
## mother        0.31761    0.03100  10.245  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.165 on 930 degrees of freedom
## Multiple R-squared:  0.6354, Adjusted R-squared:  0.6342
## F-statistic: 540.3 on 3 and 930 DF,  p-value: < 2.2e-16
```

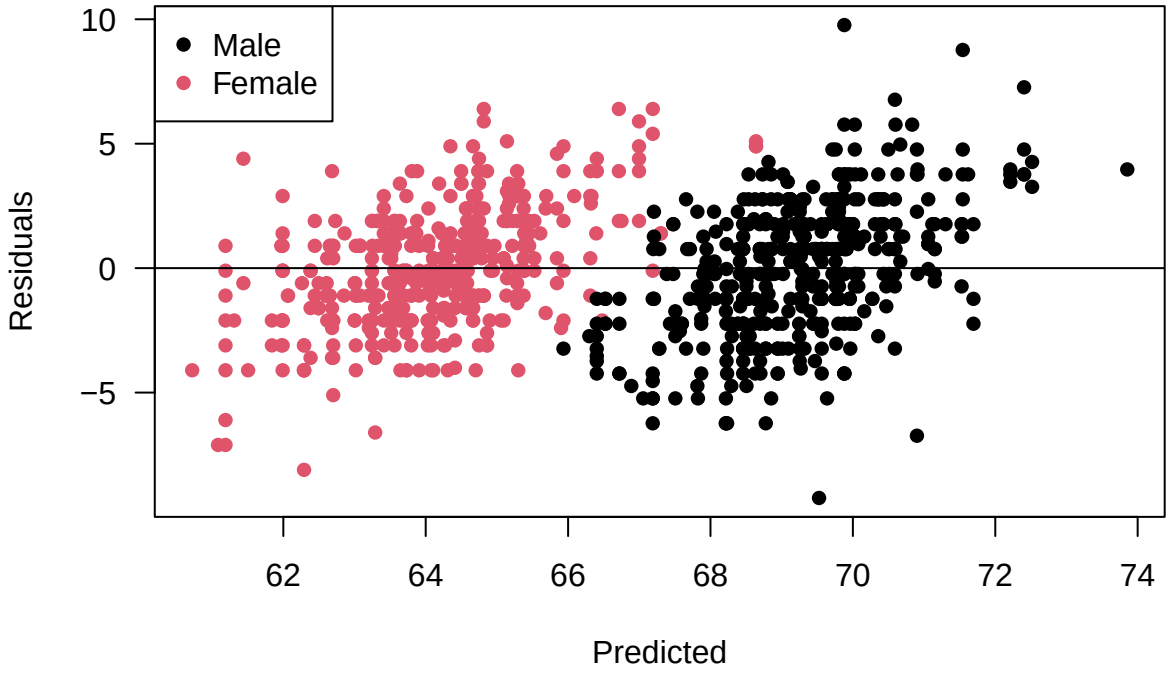
Inspect residuals. We can clearly see two clusters of points here. The cluster of points with smaller predicted heights belong to the female children and the other cluster of points belong to the male children.

```r
plot(m1$residuals ~ m1$fitted.values,
     xlab="Predicted", ylab="Residuals",
     pch=16, las=1)
abline(h=0)
```



Color-code the residuals by gender

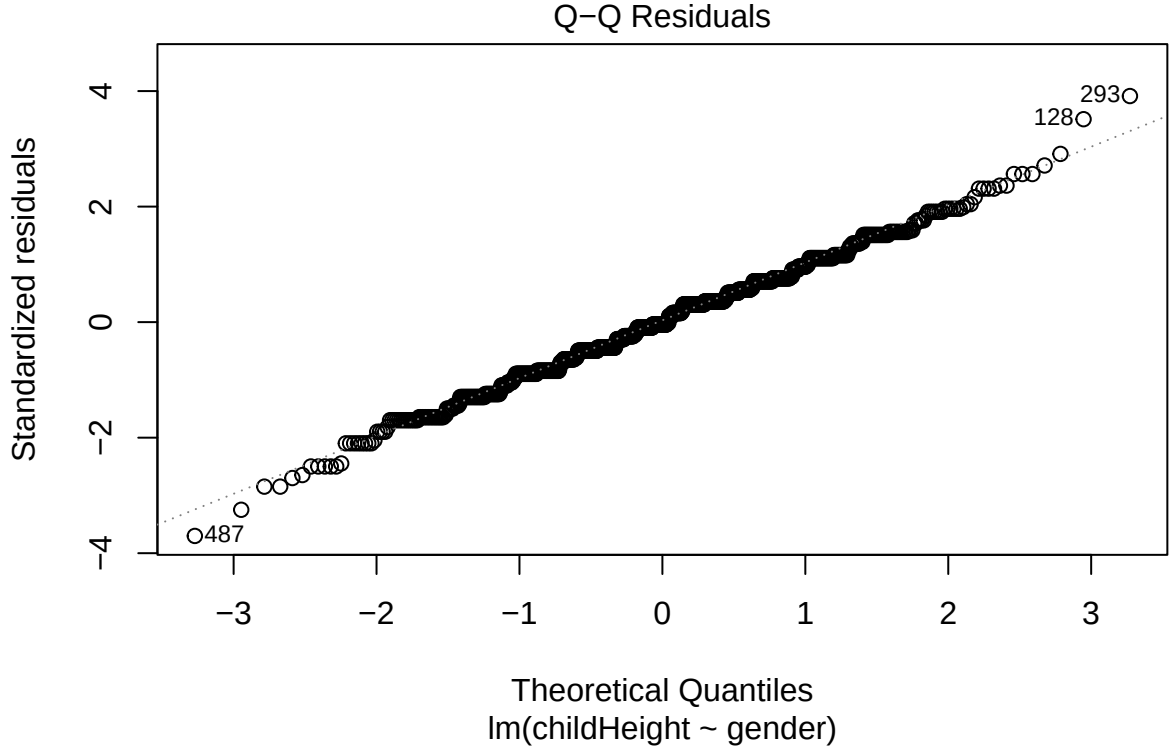
```
plot(m1$residuals ~ m2$fitted.values,
     col=GaltonFamilies$gender,
     xlab="Predicted", ylab="Residuals",
     pch=16, las=1)
abline(h=0)
legend("topleft",c("Male","Female"), pch=16, col=1:2)
```

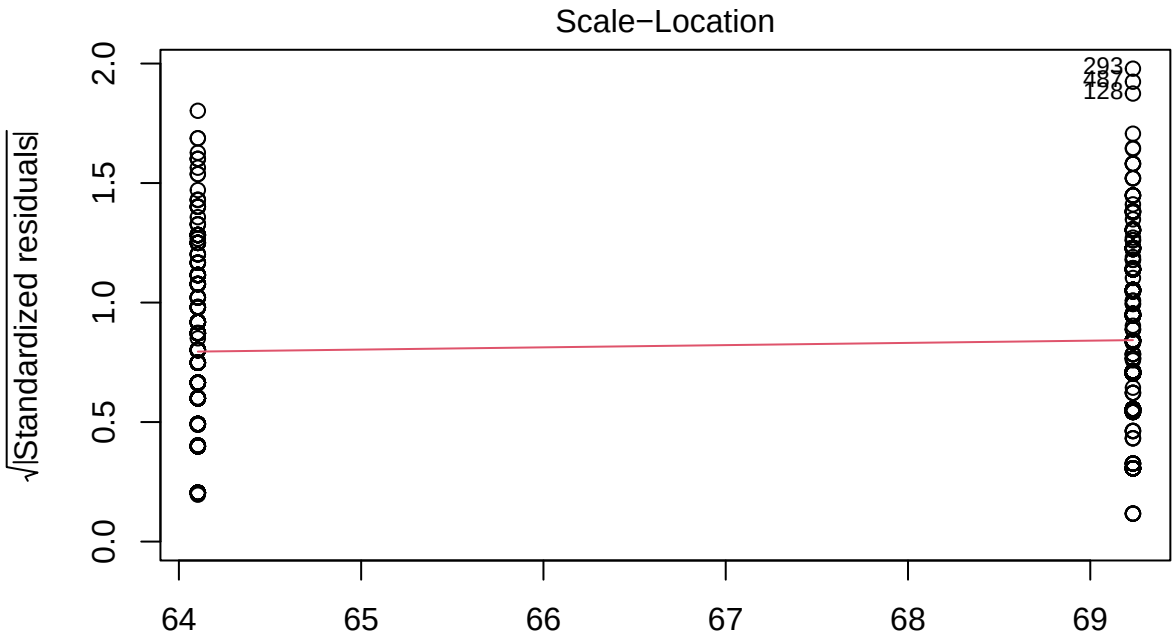


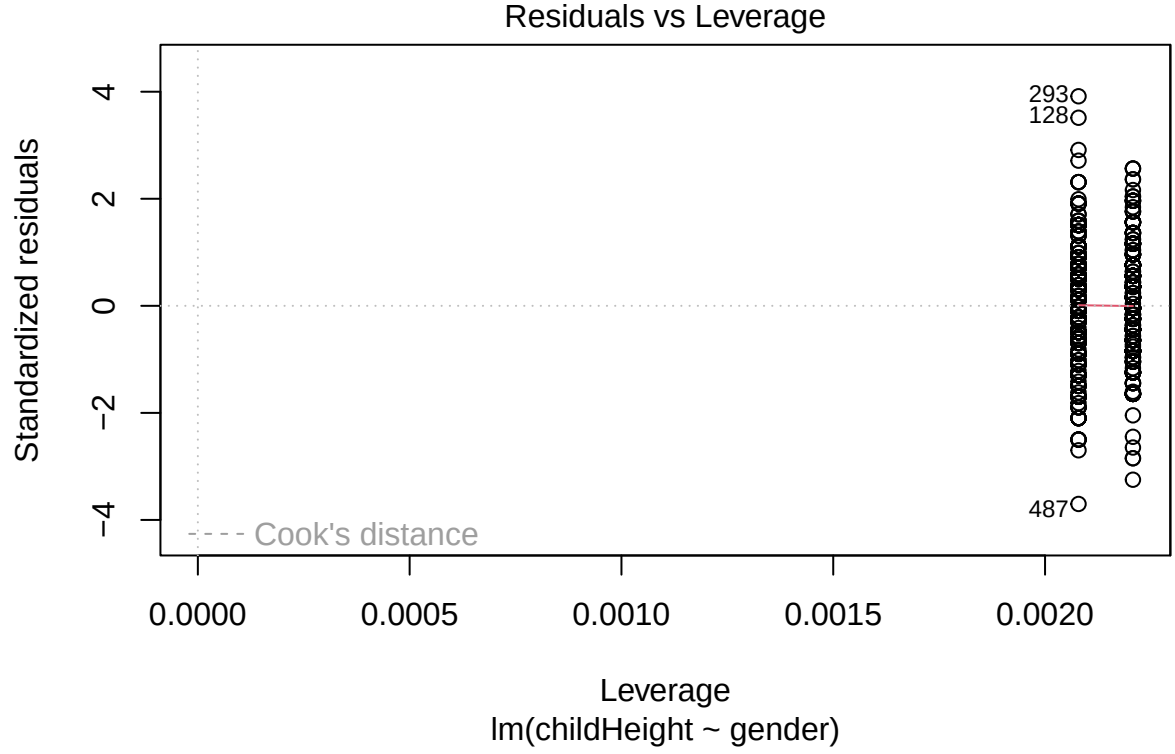
Diagnostics

```
plot(m1)
```


Residuals vs Fitted
Residuals
293
128
487
10
5
0
−5
−10
64
65
66
67
68
69
Fitted values
lm(childHeight ~ gender)


Q−Q Residuals
Standardized residuals
293
128
487
4
2
0
−2
−4
−3
−2
−1
0
1
2
3
Theoretical Quantiles
lm(childHeight ~ gender)





Scale parents. See also https://stats.stackexchange.com/questions/254934/what-is-the-interpretation-of-scaled-regression-coefficients-when-only-the-predi/254982

```
m4 = lm(scale(childHeight) ~ gender + scale(father) + scale(mother), data=GaltonFamilies)
summary(m4)
```

```
##
```

```
## Call:
## lm(formula = scale(childHeight) ~ gender + scale(father) + scale(mother),
##     data = GaltonFamilies)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.66108 -0.40938  0.02635  0.41517  2.54804
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)    0.70666    0.02758   25.62   <2e-16 ***
## genderfemale  -1.45701    0.03962  -36.77   <2e-16 ***
## scale(father)  0.27181    0.01984   13.70   <2e-16 ***
## scale(mother)  0.20329    0.01984   10.24   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6048 on 930 degrees of freedom
## Multiple R-squared:  0.6354, Adjusted R-squared:  0.6342
## F-statistic: 540.3 on 3 and 930 DF,  p-value: < 2.2e-16
```

We would expect siblings to be somewhat similar in height as they share genetic factors through their parents and environmental factors through their shared upbringing.

We can model this structure of the data, children clustering in families, using linear mixed effects models. In addition to estimating population means (fixed effects) these models will also allow us to estimate how average family heights vary around these population means (random effects).

```
library(lme4)
library(lmerTest)

# The random effect for family indicates that the mean height of each family may differ from the popula
fit_me = lmer(childHeight ~ gender + father + mother + (1|family), data=GaltonFamilies)

# In addition to the gender fixed effect that we have already seen in the simple linear regression mode
summary(fit_me)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: childHeight ~ gender + father + mother + (1 | family)
##    Data: GaltonFamilies
##
## REML criterion at convergence: 4053.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.2081 -0.5887  0.0073  0.6202  3.7316
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  family   (Intercept) 0.9073   0.9525
##  Residual             3.8197   1.9544
## Number of obs: 934, groups:  family, 205
##
## Fixed effects:
##               Estimate Std. Error        df t value Pr(>|t|)
```

```
## (Intercept)   23.56134    3.65758 193.25288   6.442 9.16e-10 ***
## genderfemale  -5.22364    0.13522 877.55415 -38.631  < 2e-16 ***
## father         0.38161    0.03798 193.65641  10.049  < 2e-16 ***
## mother         0.30180    0.04245 178.66572   7.109 2.70e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##             (Intr) gndrfm father
## genderfemal  0.021
## father      -0.670 -0.031
## mother      -0.696 -0.022 -0.065
```

```
library(MuMIn)
r.squaredGLMM(fit_me)
```

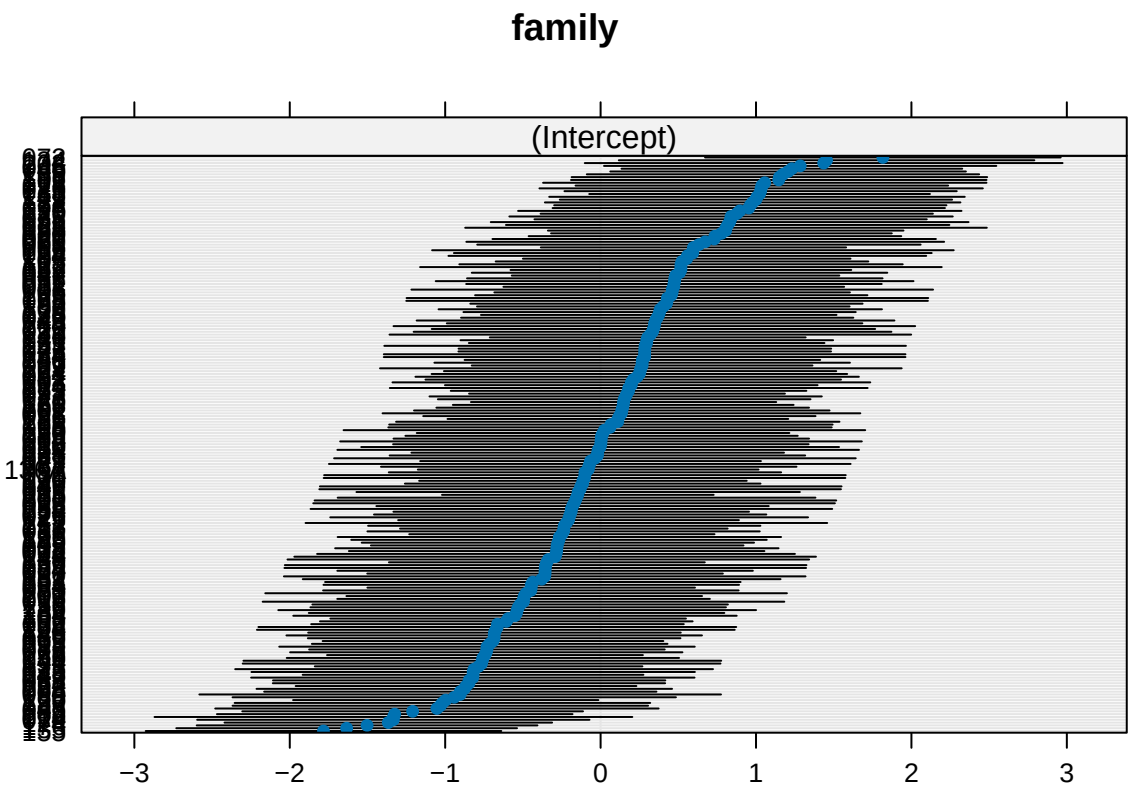
```
##            R2m       R2c
## [1,] 0.6303102 0.7012717
```

A dot plot, also known as a caterpillar plot, can help to visualise random effects. This plot shows the deviation from the mean population height for each family, together with standard errors. Note how some families fall clearly below or above the population mean.

```
library(lattice)

randoms = ranef(fit_me)
dotplot(randoms)
```

```
## $family
```



Model comparison with anova() and ranova(). In this case, the inclusion of the family random effect clearly

improves model fit.

```
fit_lm = lm(childHeight ~ gender, data=GaltonFamilies)
## Re-fit model using ML, rather than REML
fit_me = lmer(childHeight ~ gender + (1|family), data=GaltonFamilies, REML=FALSE)

anova(fit_me, fit_lm)
```

```
## Data: GaltonFamilies
## Models:
## fit_lm: childHeight ~ gender
## fit_me: childHeight ~ gender + (1 | family)
##        npar    AIC    BIC  logLik deviance  Chisq Df Pr(>Chisq)
## fit_lm    3 4364.3 4378.8 -2179.1   4358.3
## fit_me    4 4164.5 4183.8 -2078.2   4156.5 201.81  1  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
ranova(fit_me)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## childHeight ~ gender + (1 | family)
##              npar  logLik    AIC    LRT Df Pr(>Chisq)
## <none>          4 -2078.2 4164.5
## (1 | family)    3 -2179.1 4364.3 201.81  1  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Could we have modeled family as a fixed effect?

```
library(lme4)
library(lmerTest)

# The random effect for family indicates that the mean height of each family may differ from the popula
fit_me = lmer(childHeight ~ gender + father + mother + (1|family), data=GaltonFamilies)
summary(fit_me)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: childHeight ~ gender + father + mother + (1 | family)
##    Data: GaltonFamilies
##
## REML criterion at convergence: 4053.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.2081 -0.5887  0.0073  0.6202  3.7316
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  family   (Intercept) 0.9073   0.9525
##  Residual             3.8197   1.9544
## Number of obs: 934, groups:  family, 205
##
## Fixed effects:
```

```
##               Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)   23.56134    3.65758 193.25288   6.442 9.16e-10 ***
## genderfemale  -5.22364    0.13522 877.55415 -38.631  < 2e-16 ***
## father         0.38161    0.03798 193.65641  10.049  < 2e-16 ***
## mother         0.30180    0.04245 178.66572   7.109 2.70e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##             (Intr) gndrfm father
## genderfemal  0.021
## father      -0.670 -0.031
## mother      -0.696 -0.022 -0.065
```

```
fit_fe = lm(childHeight ~ family + gender + father + mother, data=GaltonFfamilies)
summary(fit_fe)
```

```
##
## Call:
## lm(formula = childHeight ~ family + gender + father + mother,
##     data = GaltonFfamilies)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.5370 -0.9955  0.0000  1.0611  6.2080
##
## Coefficients: (2 not defined because of singularities)
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)   74.0082     0.9822  75.351  < 2e-16 ***
## family002     -2.1527     1.3810  -1.559 0.119479
## family003     -1.9027     1.6912  -1.125 0.260928
## family004     -4.1816     1.3099  -3.192 0.001472 **
## family005     -4.6527     1.2608  -3.690 0.000241 ***
## family006      0.7027     2.1832   0.322 0.747625
## family007     -0.4379     1.2617  -0.347 0.728635
## family008     -0.6306     1.4916  -0.423 0.672597
## family009     -2.7973     2.1832  -1.281 0.200499
## family010     -3.2973     2.1832  -1.510 0.131396
## family011     -2.7875     1.1956  -2.331 0.020001 *
## family012     -3.7973     2.1832  -1.739 0.082397 .
## family013     -4.9027     1.6912  -2.899 0.003857 **
## family014     -6.5082     1.6943  -3.841 0.000133 ***
## family015     -2.8712     1.4924  -1.924 0.054753 .
## family016     -3.3256     1.1741  -2.833 0.004746 **
## family017     -3.9712     1.2617  -3.148 0.001714 **
## family018     -3.9639     1.4916  -2.657 0.008045 **
## family019     -6.0973     2.1832  -2.793 0.005361 **
## family020     -0.7014     1.1957  -0.587 0.557680
## family021     -0.7009     1.4912  -0.470 0.638477
## family022     -1.9379     1.4924  -1.299 0.194516
## family023     -3.0734     1.2240  -2.511 0.012257 *
## family024     -3.2973     2.1832  -1.510 0.131396
## family025     -4.2973     1.6912  -2.541 0.011262 *
## family026     -4.3238     1.3107  -3.299 0.001018 **
## family027     -5.9379     1.4924  -3.979 7.62e-05 ***
```

```
## family028     -2.2009     1.2603  -1.746 0.081180 .
## family029     -1.6639     1.4916  -1.116 0.264995
## family030     -4.7973     2.1832  -2.197 0.028306 *
## family031     -3.5694     1.2608  -2.831 0.004767 **
## family032     -2.7238     1.3107  -2.078 0.038042 *
## family033     -2.3660     1.3121  -1.803 0.071766 .
## family034     -3.0973     2.1832  -1.419 0.156412
## family035     -0.1660     1.3121  -0.127 0.899338
## family036     -2.1555     1.3824  -1.559 0.119386
## family037     -2.4750     1.3806  -1.793 0.073428 .
## family038     -4.5694     1.2608  -3.624 0.000310 ***
## family039     -2.4027     1.6912  -1.421 0.155827
## family040     -2.7660     1.3121  -2.108 0.035365 *
## family041     -3.5082     2.1855  -1.605 0.108882
## family042     -3.1046     1.2617  -2.461 0.014101 *
## family043     -2.3027     1.6912  -1.362 0.173748
## family044     -5.9082     1.6943  -3.487 0.000518 ***
## family045     -6.2712     1.4924  -4.202 2.97e-05 ***
## family046     -2.1723     1.1961  -1.816 0.069774 .
## family047     -2.5805     1.3824  -1.867 0.062357 .
## family048     -6.0082     1.4951  -4.019 6.46e-05 ***
## family049     -3.8877     1.2240  -3.176 0.001555 **
## family050     -3.9027     1.6912  -2.308 0.021298 *
## family051     -2.7973     1.6912  -1.654 0.098561 .
## family052     -4.3816     1.3099  -3.345 0.000865 ***
## family053     -5.0021     1.1736  -4.262 2.29e-05 ***
## family054     -3.8305     1.3824  -2.771 0.005734 **
## family055     -4.9816     1.3099  -3.803 0.000155 ***
## family056     -4.4238     1.3107  -3.375 0.000777 ***
## family057     -4.9395     1.3097  -3.771 0.000176 ***
## family058     -4.9480     1.2256  -4.037 5.98e-05 ***
## family059     -6.7973     2.1832  -3.114 0.001921 **
## family060     -3.7582     1.6943  -2.218 0.026851 *
## family061     -2.9555     1.3824  -2.138 0.032859 *
## family062     -4.0842     1.2603  -3.241 0.001247 **
## family063      0.9918     2.1855   0.454 0.650117
## family064     -5.0816     1.3099  -3.879 0.000114 ***
## family065     -1.2973     2.1832  -0.594 0.552555
## family066     -2.8467     1.1401  -2.497 0.012747 *
## family067     -4.0832     1.3848  -2.949 0.003294 **
## family068     -5.4973     1.3102  -4.196 3.06e-05 ***
## family069     -5.1791     1.1968  -4.327 1.72e-05 ***
## family070     -6.5816     1.3099  -5.025 6.35e-07 ***
## family071     -3.8676     1.2603  -3.069 0.002230 **
## family072     -1.2162     1.2240  -0.994 0.320718
## family073     -2.1046     1.4924  -1.410 0.158905
## family074     -5.0082     1.6943  -2.956 0.003218 **
## family075     -3.6162     1.2240  -2.954 0.003233 **
## family076     -5.5924     1.2269  -4.558 6.05e-06 ***
## family077     -5.1555     1.3824  -3.729 0.000207 ***
## family078     -5.7216     1.3099  -4.368 1.44e-05 ***
## family079     -4.3305     1.1978  -3.615 0.000320 ***
## family080     -8.7973     2.1832  -4.030 6.17e-05 ***
## family081     -5.3500     1.3806  -3.875 0.000116 ***
```

```
## family082     -3.7157     1.1748  -3.163 0.001627 **
## family083     -6.3639     1.1957  -5.322 1.37e-07 ***
## family084     -4.5277     1.3810  -3.279 0.001093 **
## family085     -4.5238     1.3107  -3.452 0.000590 ***
## family086     -4.0277     1.3810  -2.916 0.003649 **
## family087     -6.2277     1.3810  -4.509 7.57e-06 ***
## family088     -6.5277     1.3810  -4.727 2.74e-06 ***
## family089     -4.4291     1.1968  -3.701 0.000231 ***
## family090     -5.2750     1.2246  -4.307 1.88e-05 ***
## family091     -4.2712     1.4924  -2.862 0.004330 **
## family092     -4.9082     1.6943  -2.897 0.003881 **
## family093     -6.5277     1.3810  -4.727 2.74e-06 ***
## family094     -3.7973     1.6912  -2.245 0.025049 *
## family095     -5.9379     1.4924  -3.979 7.62e-05 ***
## family096     -4.4816     1.3099  -3.421 0.000658 ***
## family097     -3.2105     1.1551  -2.780 0.005585 **
## family098     -4.7973     2.1832  -2.197 0.028306 *
## family099     -3.4152     1.1961  -2.855 0.004423 **
## family100     -3.2749     1.4951  -2.190 0.028809 *
## family101     -1.6332     1.3848  -1.179 0.238621
## family102     -5.2361     1.2608  -4.153 3.67e-05 ***
## family103     -4.8352     1.2269  -3.941 8.89e-05 ***
## family104     -4.7777     1.3810  -3.460 0.000573 ***
## family105     -3.1658     1.2603  -2.512 0.012225 *
## family106     -3.2750     1.2246  -2.674 0.007657 **
## family107     -3.3910     1.1736  -2.889 0.003974 **
## family108     -5.4591     1.2240  -4.460 9.49e-06 ***
## family109     -6.5877     1.2240  -5.382 9.94e-08 ***
## family110     -4.7277     1.3810  -3.423 0.000653 ***
## family111     -3.2973     2.1832  -1.510 0.131396
## family112     -3.8676     1.4912  -2.594 0.009690 **
## family113     -2.0082     2.1855  -0.919 0.358465
## family114     -4.7712     1.2617  -3.782 0.000169 ***
## family115     -6.1020     1.2240  -4.985 7.74e-07 ***
## family116     -4.8009     1.4912  -3.219 0.001341 **
## family117     -6.2973     2.1832  -2.884 0.004036 **
## family118     -2.6749     1.4951  -1.789 0.074011 .
## family119     -3.5660     1.3121  -2.718 0.006728 **
## family120     -5.0184     1.1400  -4.402 1.23e-05 ***
## family121     -5.0902     1.1961  -4.256 2.36e-05 ***
## family122     -3.4027     1.3810  -2.464 0.013973 *
## family123     -5.2238     1.3107  -3.986 7.41e-05 ***
## family124     -7.7157     1.1748  -6.568 9.72e-11 ***
## family125     -3.5342     1.4912  -2.370 0.018046 *
## family126     -7.1027     1.3810  -5.143 3.48e-07 ***
## family127     -4.5082     2.1855  -2.063 0.039489 *
## family128     -5.0527     1.6912  -2.988 0.002906 **
## family129     -1.9379     1.4924  -1.299 0.194516
## family130     -5.6163     1.1439  -4.910 1.13e-06 ***
## family131     -7.2582     1.6943  -4.284 2.08e-05 ***
## family132     -6.4027     1.6912  -3.786 0.000166 ***
## family133     -4.2607     1.2246  -3.479 0.000533 ***
## family134     -1.9027     1.3810  -1.378 0.168697
## family135     -7.1902     1.1961  -6.011 2.91e-09 ***
```

```
## family136     -5.6816     1.1553  -4.918 1.08e-06 ***
## family136A    -4.8166     1.1968  -4.025 6.31e-05 ***
## family137     -7.2777     1.3810  -5.270 1.80e-07 ***
## family138     -4.4860     1.3121  -3.419 0.000664 ***
## family139     -6.7973     2.1832  -3.114 0.001921 **
## family140     -4.7605     1.1551  -4.121 4.20e-05 ***
## family141     -5.8041     1.1968  -4.850 1.51e-06 ***
## family142     -3.0805     1.3824  -2.228 0.026165 *
## family143     -7.0082     2.1855  -3.207 0.001402 **
## family144     -4.7777     1.3810  -3.460 0.000573 ***
## family145     -6.7416     1.1968  -5.633 2.53e-08 ***
## family146     -6.6527     1.2608  -5.277 1.74e-07 ***
## family147     -5.8082     2.1855  -2.658 0.008043 **
## family148     -4.0082     2.1855  -1.834 0.067064 .
## family149     -5.7660     1.3121  -4.394 1.28e-05 ***
## family150     -5.5082     2.1855  -2.520 0.011938 *
## family151     -6.7027     1.6912  -3.963 8.12e-05 ***
## family152     -7.5082     2.1855  -3.435 0.000625 ***
## family153     -6.5238     1.3107  -4.977 8.05e-07 ***
## family154     -7.3082     2.1855  -3.344 0.000868 ***
## family155     -9.2560     1.2238  -7.563 1.19e-13 ***
## family156     -7.4555     1.3824  -5.393 9.38e-08 ***
## family157     -5.0082     2.1855  -2.292 0.022217 *
## family158     -7.7538     1.1562  -6.707 4.00e-11 ***
## family159     -2.7438     1.3107  -2.093 0.036655 *
## family160     -3.0082     2.1855  -1.376 0.169112
## family161     -3.4041     1.1968  -2.844 0.004576 **
## family162     -4.5842     1.2603  -3.637 0.000295 ***
## family163     -4.2816     1.3099  -3.269 0.001131 **
## family164     -6.2277     1.3810  -4.509 7.57e-06 ***
## family165     -7.5342     1.4912  -5.052 5.52e-07 ***
## family166     -5.4265     1.1400  -4.760 2.33e-06 ***
## family167     -5.1332     1.3848  -3.707 0.000226 ***
## family168     -5.4944     1.1990  -4.582 5.41e-06 ***
## family169     -5.6046     1.4924  -3.755 0.000187 ***
## family170     -4.3660     1.3121  -3.327 0.000920 ***
## family171     -7.0082     2.1855  -3.207 0.001402 **
## family172     -6.3666     1.1968  -5.320 1.39e-07 ***
## family173     -6.2009     1.1733  -5.285 1.66e-07 ***
## family174     -4.8816     1.3099  -3.727 0.000209 ***
## family175     -4.8676     1.2603  -3.862 0.000122 ***
## family176     -7.7527     1.1961  -6.481 1.67e-10 ***
## family177     -3.7238     1.3107  -2.841 0.004621 **
## family178     -4.0082     2.1855  -1.834 0.067064 .
## family179     -5.5473     1.6912  -3.280 0.001087 **
## family180     -6.0361     1.2608  -4.788 2.05e-06 ***
## family181     -5.7861     1.2237  -4.728 2.72e-06 ***
## family182     -4.0082     2.1855  -1.834 0.067064 .
## family183     -7.7055     1.3824  -5.574 3.51e-08 ***
## family184     -9.0082     2.1855  -4.122 4.19e-05 ***
## family185     -7.6490     1.0991  -6.959 7.64e-12 ***
## family186     -6.8305     1.3824  -4.941 9.66e-07 ***
## family187     -5.7973     2.1832  -2.655 0.008093 **
## family188     -7.8500     1.3806  -5.686 1.88e-08 ***
```

```
## family189     -7.1238     1.3107  -5.435 7.47e-08 ***
## family190     -6.2009     1.1733  -5.285 1.66e-07 ***
## family191     -3.3027     1.6912  -1.953 0.051217 .
## family192     -5.7879     1.2617  -4.587 5.28e-06 ***
## family193     -6.3676     1.2603  -5.052 5.52e-07 ***
## family194     -4.9027     1.6912  -2.899 0.003857 **
## family195     -7.2712     1.4924  -4.872 1.35e-06 ***
## family196     -4.0805     1.3824  -2.952 0.003262 **
## family197     -6.5660     1.3121  -5.004 7.04e-07 ***
## family198     -5.4290     1.2237  -4.436 1.06e-05 ***
## family199     -4.7861     1.2237  -3.911 0.000100 ***
## family200     -9.5082     2.1855  -4.351 1.55e-05 ***
## family201     -8.4027     1.6912  -4.968 8.42e-07 ***
## family202     -2.7973     1.6912  -1.654 0.098561 .
## family203     -8.2009     1.4912  -5.499 5.28e-08 ***
## family204     -9.6527     1.6912  -5.708 1.67e-08 ***
## genderfemale  -5.2110     0.1442 -36.137  < 2e-16 ***
## father             NA         NA      NA       NA
## mother             NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.952 on 728 degrees of freedom
## Multiple R-squared:  0.7678, Adjusted R-squared:  0.7025
## F-statistic: 11.74 on 205 and 728 DF,  p-value: < 2.2e-16
```

```
anova(fit_me, fit_fe)
```

```
## Data: GaltonFamilies
## Models:
## fit_me: childHeight ~ gender + father + mother + (1 | family)
## fit_fe: childHeight ~ family + gender + father + mother
##        npar    AIC    BIC  logLik deviance  Chisq  Df Pr(>Chisq)
## fit_me    6 4050.9 4080.0 -2019.5   4038.9
## fit_fe  207 4081.7 5083.4 -1833.8   3667.7 371.27 201  2.948e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```