Network Analysis:

The Hidden Structures behind the Webs We Weave

17-213 / 17-668

Edges vs. Social Ties

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2-min Quiz, on Canvas

Quick Recap – Last Thursday's Lecture

Graph component and dyad shortest path both use breadth-first search (BFS)

Random network models: Useful baseline model

- N (# nodes), p (tie probability) → L (# edges) and $<k>$ (mean degree)
- Critical point at which a giant component forms → $<k> > 1$
- Average path length grows slower than the growth of a network ~ ln(N)
- Hence the small-world

Triad is the smallest social group

Structural balance theory (Friend of my friend, enemy of my enemy…)

- Triad is unbalanced with odd number of negative edges

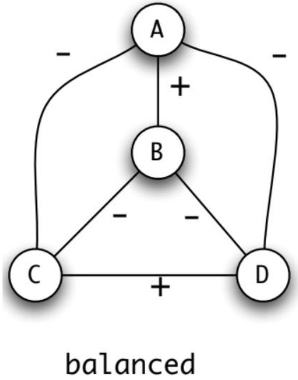
Structural Balance: Triads of Friends and Enemies

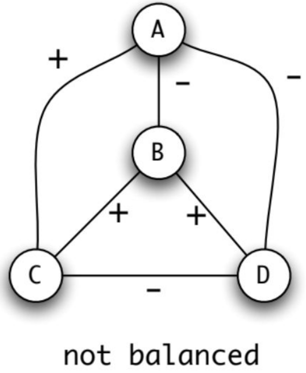
We can extend this logic of balance in triads to an entire graph

→Are all triads in a graph structurally balanced?

→ Does every triad have an even number of negative ties?

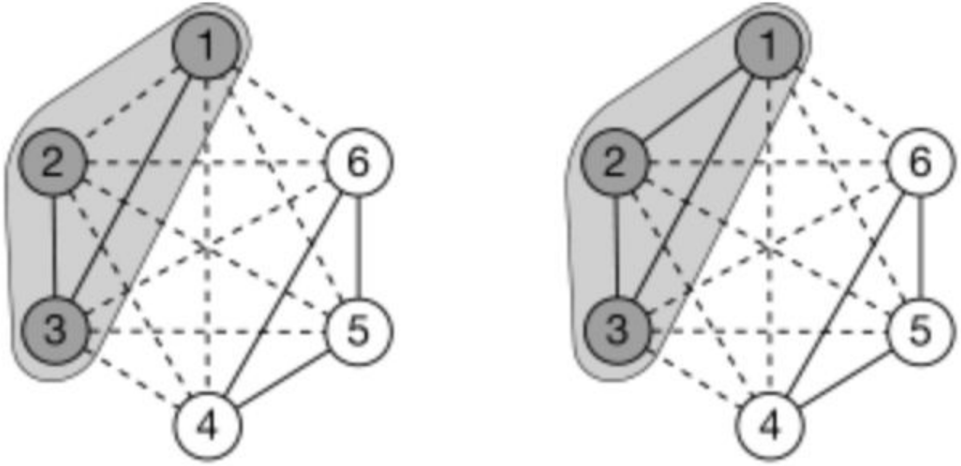
A graph with even a single unbalanced triad is “unbalanced”





Structural Balance: Triads of Friends and Enemies

For these completely connected graphs with positive (solid) and negative (dashed) edges, which one is balanced?



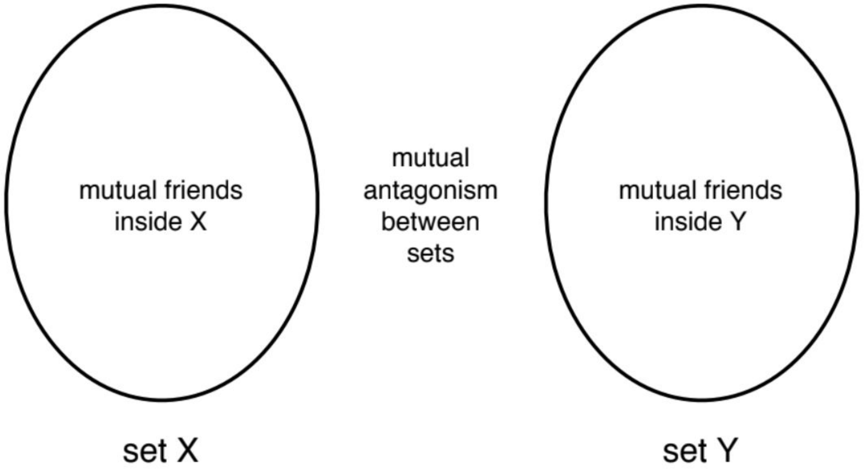
Structural Balance: Triads of Friends and Enemies

A completely connected graph is balanced iff:

(a) all pairs of nodes are friends

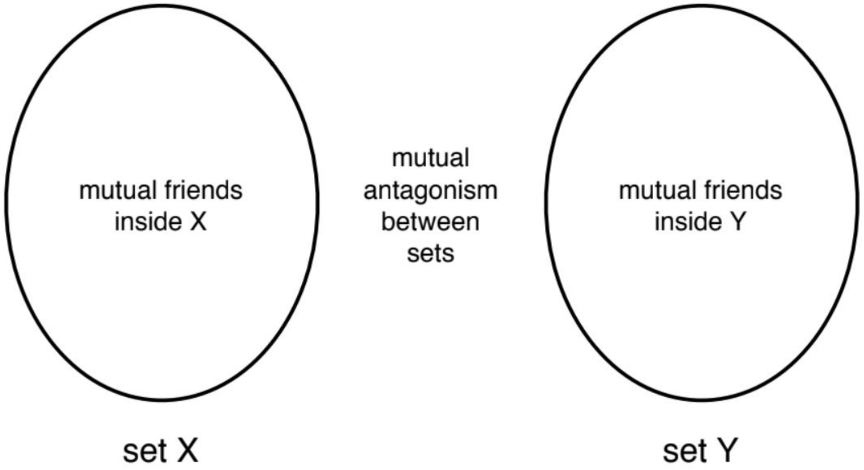
or

(b) all nodes can be divided into two groups X and Y where everyone in the same group are friends and all the pairs between groups are enemies



Structural Balance: Triads of Friends and Enemies

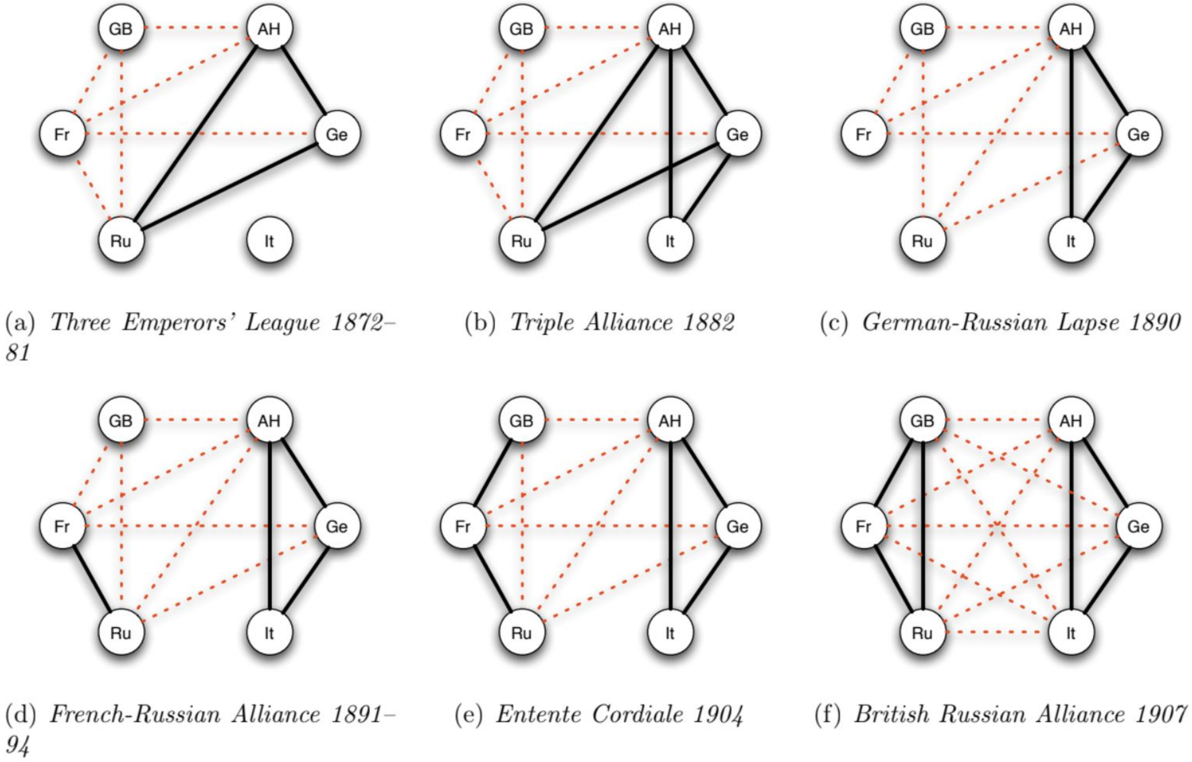
A completely connected, structurally balanced graph implies perfect polarization



Structural Balance: Triads of Friends and Enemies

Structural balance in the real world:

The evolution of alliances among nations in 19th century Europe



Solid lines: Allies, Dashed lines: Enemies

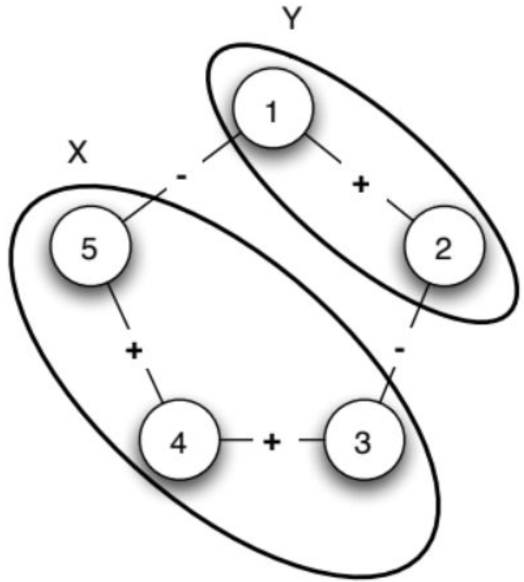
Structural Balance: Triads of Friends and Enemies

What about graphs that are not completely connected?

Balance can be defined for signed graphs in general as the following:

Nodes can be divided into two groups X and Y where all the edges in X are positive, all edges in Y are positive, and all edges between X and Y are negative

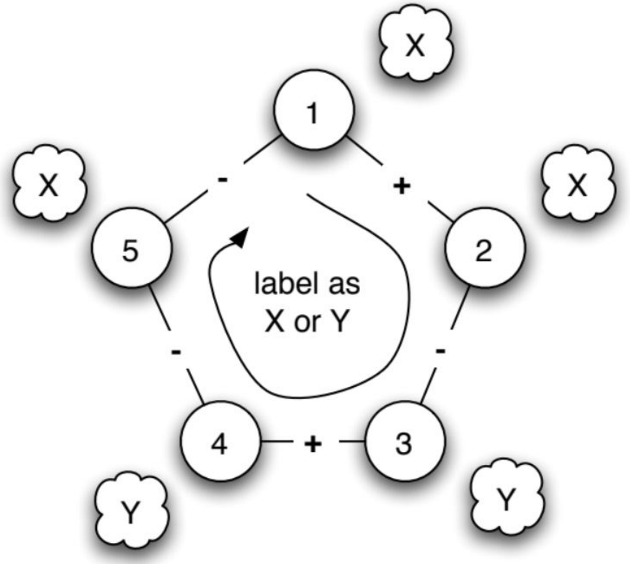
(Not all dyads need to be connected)



Structural Balance: Triads of Friends and Enemies

How to checking for balance:

Search for a cycle of any length that has an odd number of negative edges → Unbalanced



Start from a starting node and label the adjacent node according to the sign of the edge

Structural Balance: Triads of Friends and Enemies

Structural balance is an “ideal type,” a model against which empirical social networks can be evaluated

Measures of structural balance

- **Line index of balance**: How far away is a given graph from perfect balance?
- The minimum number of edges whose signs need to be changed to obtain perfect balance

- **Cycle index for balance**: Proportion of positive cycles in a signed graph

Structural Balance: Triads of Friends and Enemies

Balance theory is a useful starting point for understanding factions in small groups, group structure, and polarization

As we will see in later lectures, many models of political polarization are based, at their core, on the logic of structural balance

But, most real world social networks are not perfectly balanced.

The Reality is More Complex

Social networks in the wild:

- Far from perfect structural balance
- Signed relationships are rare

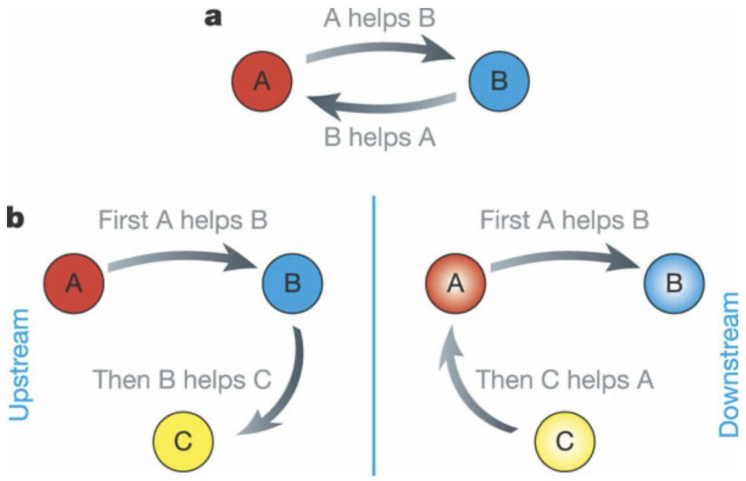
  → usually, presence/absence of an edge is more adequate

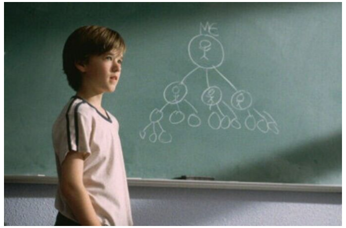
- Many different triadic relationships exist. For example:
  - hierarchical pattern (a→b→c)
  - Cyclical pattern (a→b→c→a)

An Example of Three-Way Relations: Reciprocity

Example: direct reciprocity (a) vs. generalized reciprocity (b)

- Generalized reciprocity is reciprocity in one direction
- The receiver does not give back to the giver, but **pays it forward**
- Represented as **cycles** (what goes around comes around)
- Such a configuration can require high levels of trust among group members



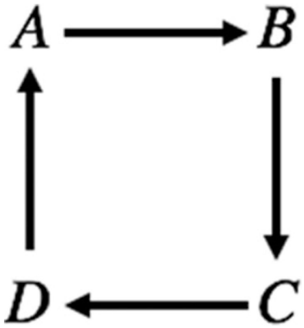


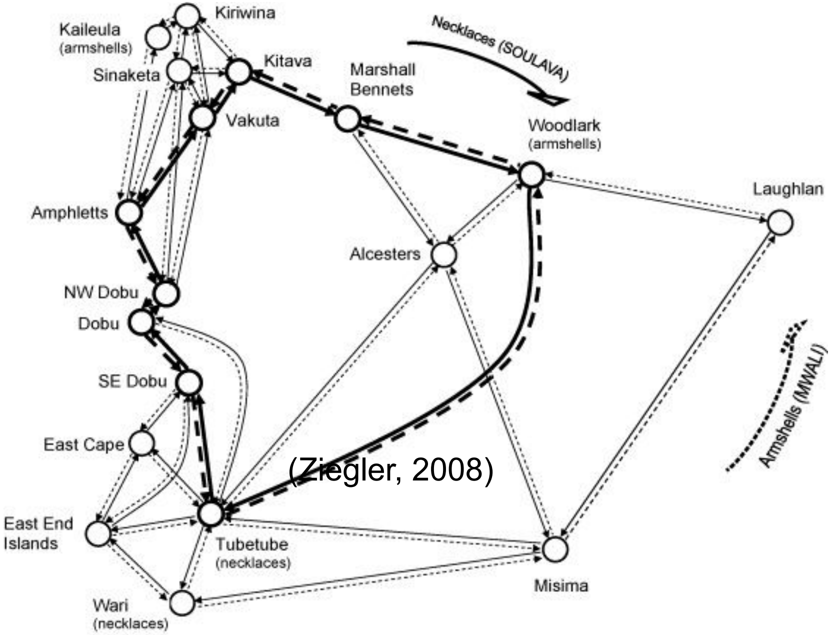
Generalized Reciprocity

A more flexible triad representation is needed to study diverse network phenomena

Example:

Systems of generalized reciprocity: Kula ring



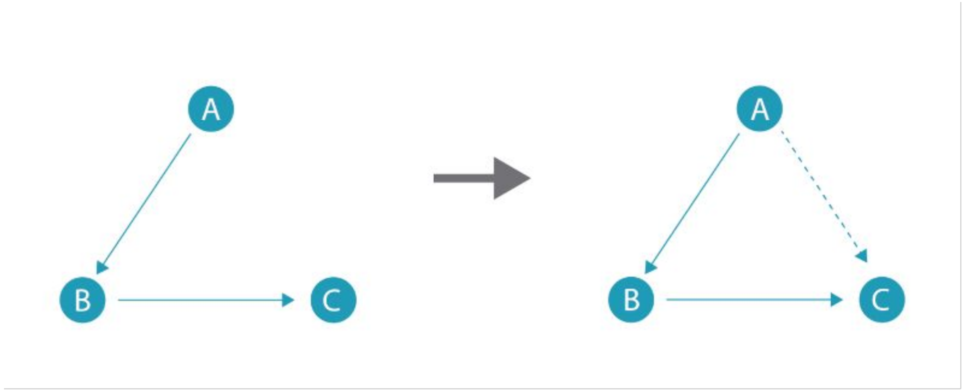


(Ziegler, 2008)

Transitivity

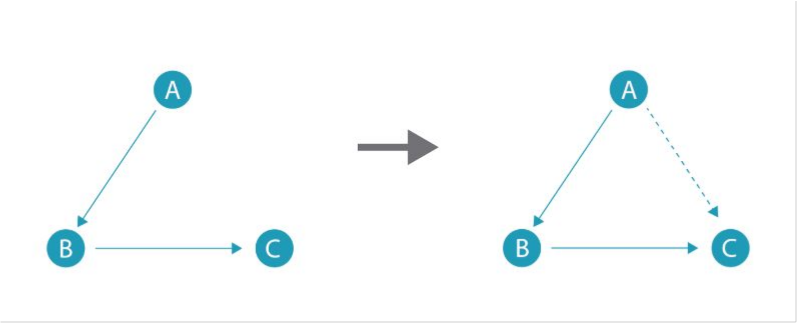
A prevalent triadic pattern in most social networks is transitivity

Triad of A, B, and C is transitive if whenever **A→B** *and* **B→C**, then **A→C**



Transitivity

Transitivity is the most basic structural representation of hierarchy and dominance orders



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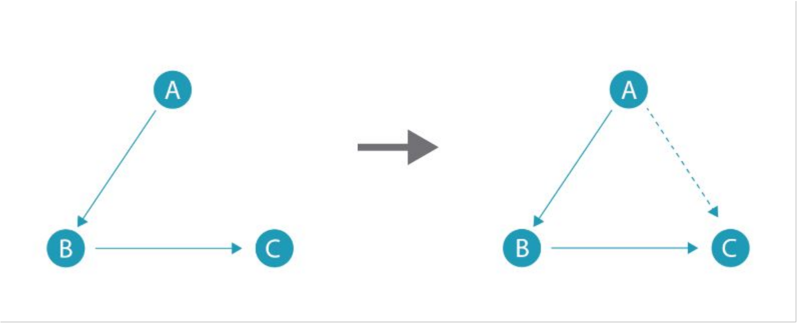
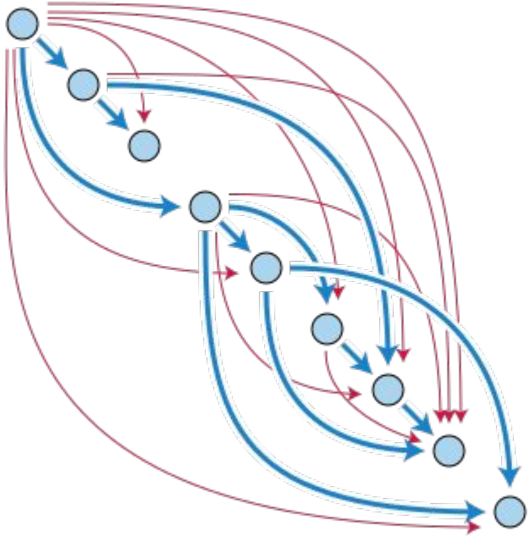


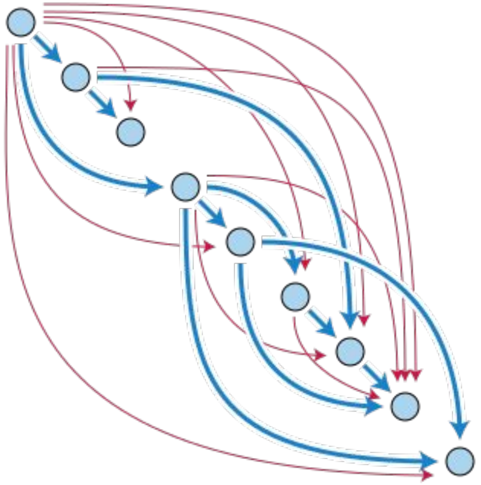


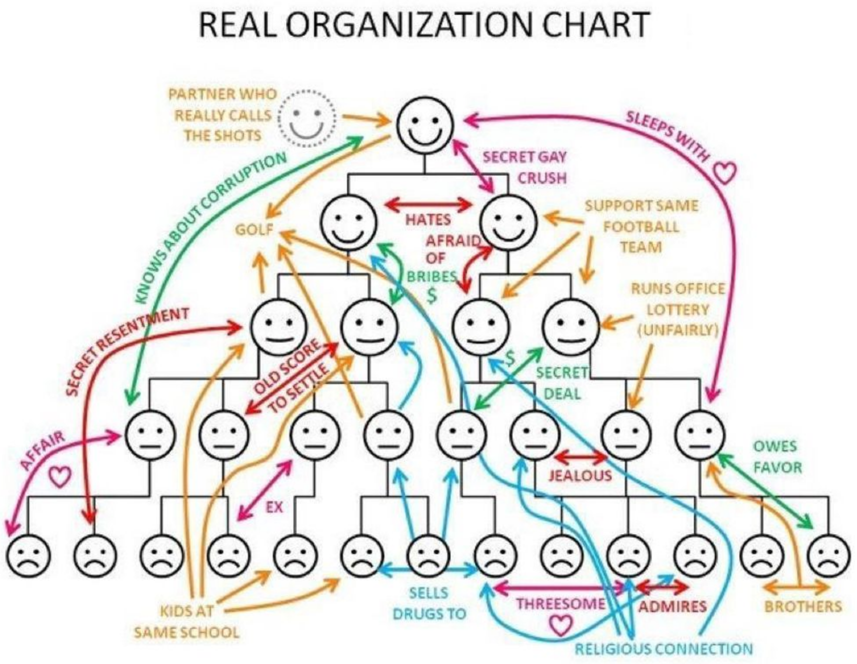
It is the building block of larger transitive networks

Transitivity

Transitivity is also an ideal type

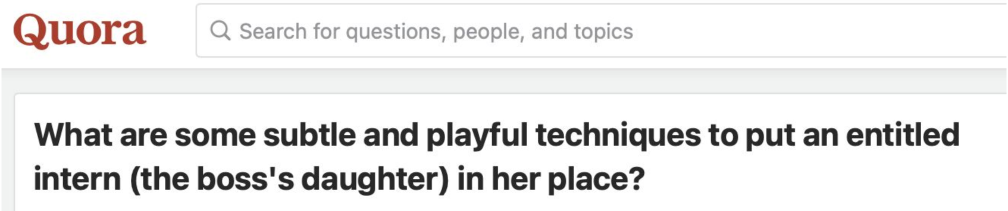
Perfectly transitive networks are rare

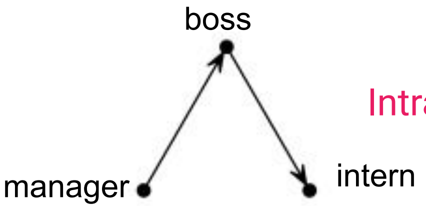




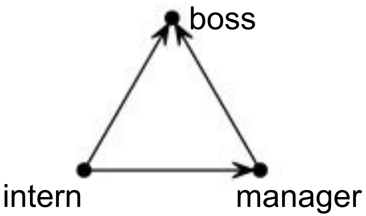
Transitivity

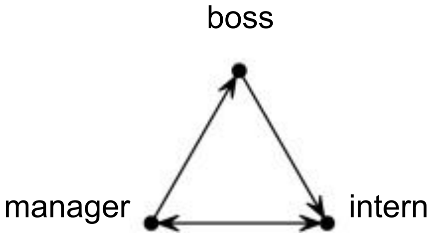
We know when a situation deviates from perfect transitivity





Intransitive → Transitive

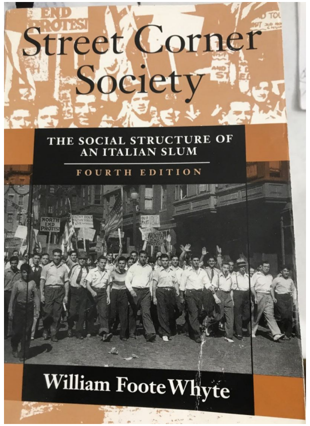


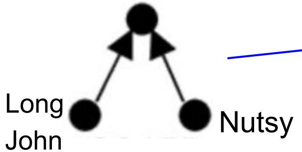


Transitivity

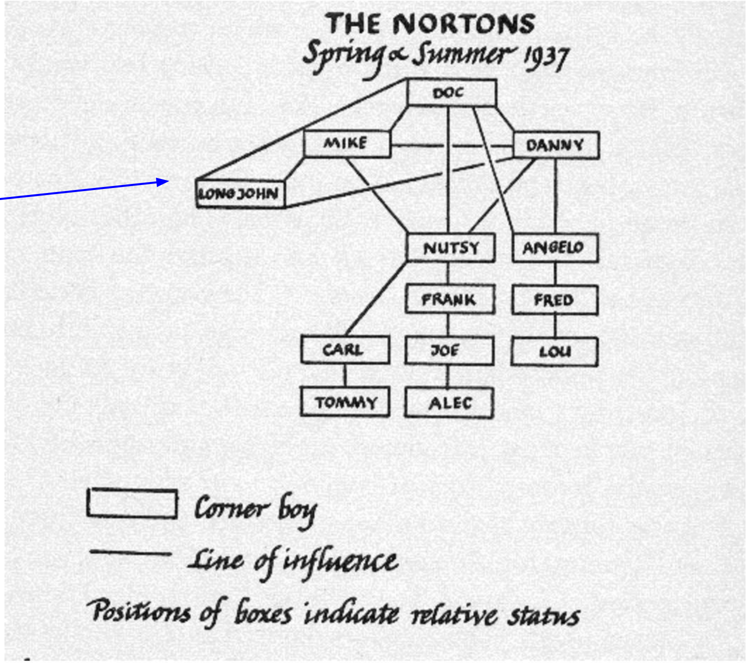
An individual's status tends to be fragile without transitivity

Long John is high in status because of his connections to other high status members, but without the connections to the lower ranks





intransitive



Triad Census

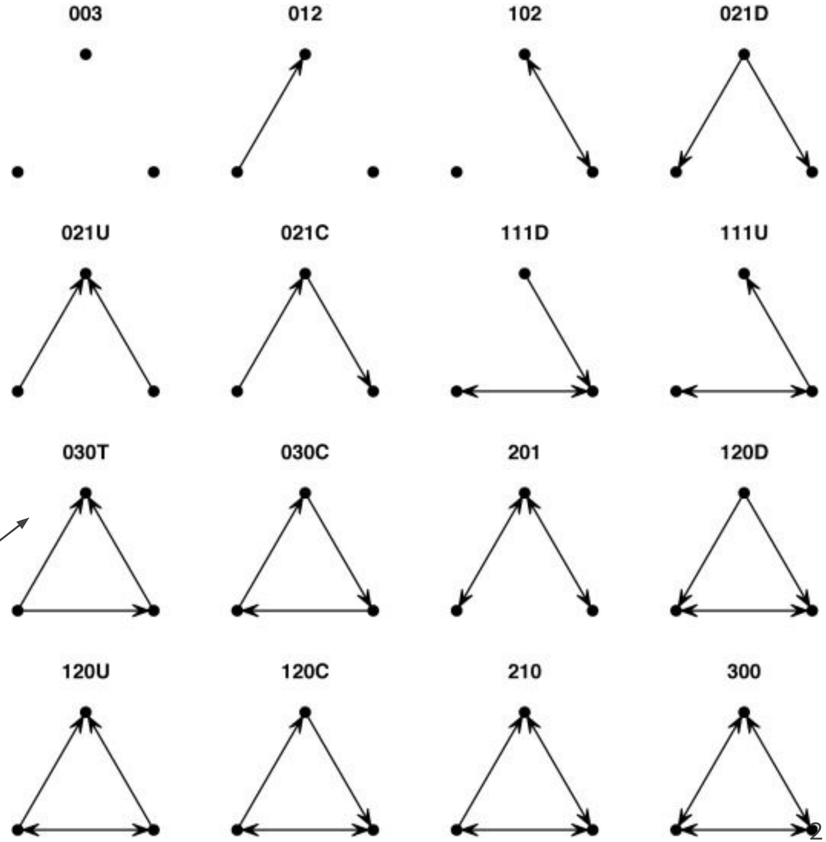
Cycles and transitive triads are part of 16 triad **isomorphism classes**

Built on combinations of dyad isomorphism classes (M, A, N)

Class names are based on the number of M, A, N dyads

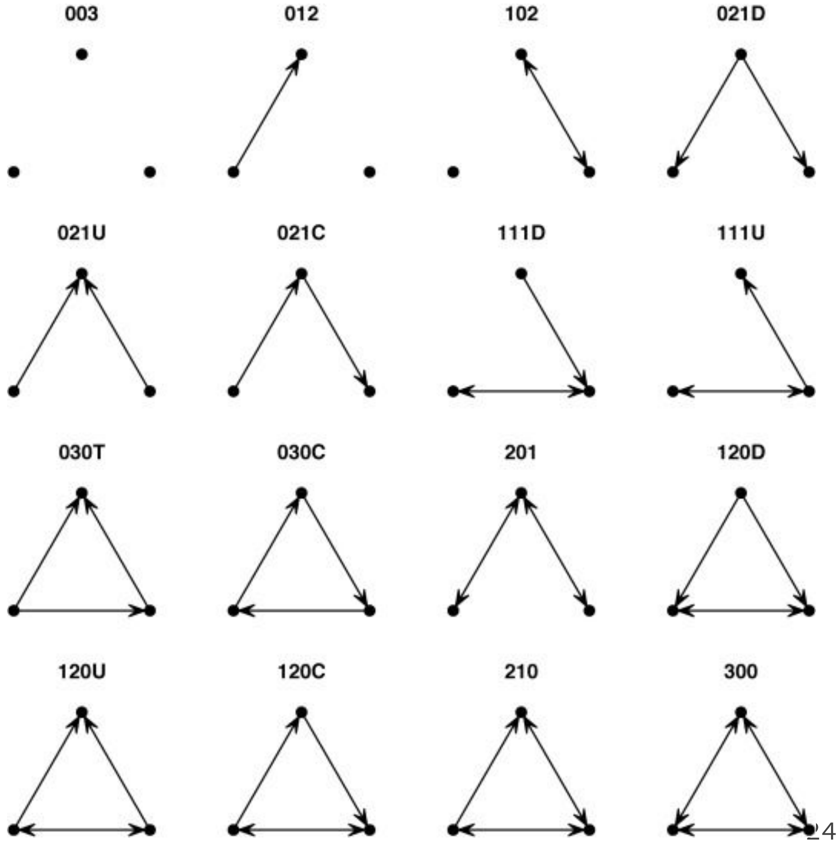
Triad census counts the frequency of these 16 isomorphism classes in a network

Mutual: 0, Asymmetric: 3, Null: 0



Triad Census

Just like the dyad census (M, A, N dyads), the triad census can give clues to the **social forces / mechanisms** that may be prevalent in a given social network

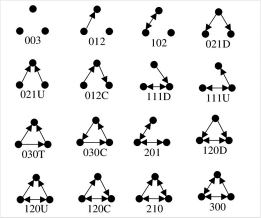


Triad Census

Just like the dyad census (M, A, N dyads), the triad census can give clues to the **social forces / mechanisms** that may be prevalent in a given social network

Example:

Networks where the 030T triads are prevalent are probably hierarchical in nature



Triad type	Triad census	Expected value	Standard deviation
003	376	320.06	9.39
012	366	416.82	14.56
102	143	171.19	9.43
021*D*	114	44.09	6.22
021*U*	34	44.09	6.22
021*C*	35	88.17	8.17
111*D*	39	73.74	7.78
111*U*	101	73.74	7.78
030*T*	23	18.17	3.86
030*C*	0	6.06	2.39
201	20	28.97	4.52
120*D*	16	7.74	2.71
120*U*	25	7.74	2.71
120*C*	9	15.48	3.74
210	23	12.38	3.25

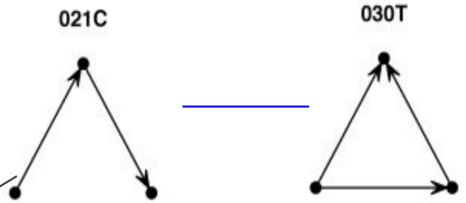
Triad Census

We can also study what structural tendencies characterize a network by looking at the covariance / correlation between different isomorphism classes

Which classes show high correlation and why?

Covariance matrix of a friendship network

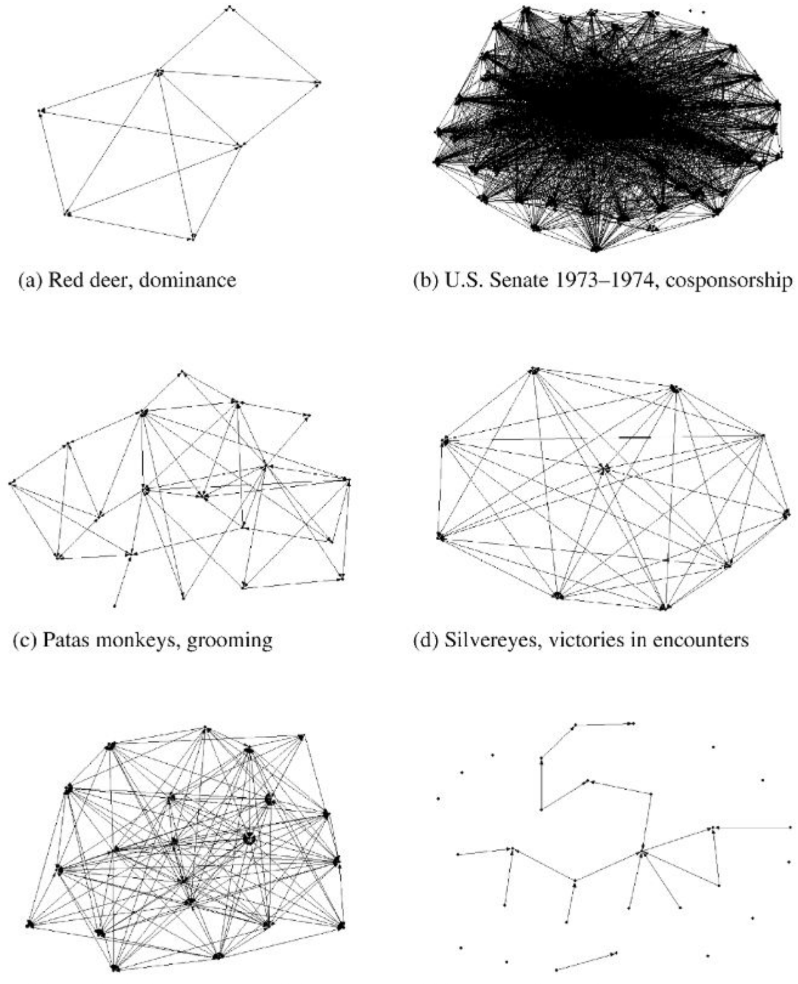
	003	012	102	021D	021U	021C	111D	111U	030T	030C	201	120D	120U	120C	210	300
003	88.2															
012	-107.0	212.0														
102	-44.0	-3.24	88.9													
021D	4.71	-21.4	4.55	38.7												
021U	4.71	-21.4	4.55	-5.35	38.7											
021C	9.42	-42.9	9.11	-10.7	-10.7	66.8										
111D	7.87	-8.65	-19.6	-2.09	-2.09	-4.18	60.5									
111U	7.87	-8.65	-19.6	-2.09	-2.09	-4.18	-13.3	60.5								
030T	6.65	-5.68	3.56	-3.57	-3.57	$\boxed{-7.14}$	-1.01	-1.01	14.9							
030C	2.22	-1.89	1.19	-1.19	-1.19	-2.38	-0.34	-0.34	-1.09	-5.69						
201	3.09	7.29	-18.4	1.12	1.12	2.23	-6.02	-6.02	0.60	0.20	20.5					
120D	2.83	-0.38	-0.52	-0.68	-0.68	-1.35	-1.77	-1.77	-0.63	-0.21	-0.43	7.36				
120U	2.83	-0.38	-0.52	-0.68	-0.68	-1.35	-1.77	-1.77	-0.63	-0.21	-0.43	-0.38	7.36			
120C	5.66	-0.77	-1.04	-1.35	-1.35	-2.71	-3.54	-3.54	-1.25	-0.42	-0.86	-0.76	-0.76	14.0		
210	4.53	2.64	-4.09	-0.06	-0.06	-0.13	-3.66	-3.66	-0.17	-0.06	-3.11	-0.58	-0.58	-1.17	10.5	
300	0.57	0.74	-0.92	0.08	0.08	0.16	-0.40	-0.40	-0.03	-0.01	-0.86	-0.04	-0.04	-0.08	-0.37	1.44



Triad Census

One can gain insight about a community or social group by **comparing the distributions** of the 16 isomorphism classes across different networks

But, direct comparison of the observed isomorphism classes is problematic because networks have different size, density, and dyad distributions (M, A, N)



(a) Red deer, dominance

(b) U.S. Senate 1973–1974, cosponsorship

(c) Patas monkeys, grooming

(d) Silvereyes, victories in encounters

(e) Krackhardt's managers, advice

(f) Cows, social licking

Triad Census

An apples-to-apples comparison involves comparing the empirical network to a random graph of the same size and same number of dyad isomorphism classes

By quantifying the deviation from random, we can compare different networks through this deviation

- Interpretation: Relative to a random network, observed network deviates by X amount

random observed

Triad Type	Formula	34 Group		35 Group		44 Group		45 Group	
		Theor. (%)	Real (%)	Theor. (%)	Real (%)	Theor. (%)	Real (%)	Theor. (%)	Real (%)
003	$(1-\Delta)^6$	0.15	1.05	0.00	0.00	17.80	29.26	26.02	38.00
012	$6\Delta(1-\Delta)^5$	1.80	2.10	0.12	0.29	35.60	23.97	39.27	25.94
102	$3\Delta^2(1-\Delta)^4$	1.75	2.45	0.26	1.62	5.93	12.79	4.94	11.95
021D	$3\Delta^2(1-\Delta)^4$	1.75	9.09	0.26	0.74	5.93	6.47	4.94	5.94
021U	$3\Delta^2(1-\Delta)^4$	1.75	1.40	0.26	0.59	5.93	3.24	4.94	2.44
021C	$6\Delta^2(1-\Delta)^4$	3.49	0.00	0.52	0.15	11.87	1.91	9.88	1.06
111D	$6\Delta^3(1-\Delta)^3$	6.78	0.35	2.21	2.21	3.96	0.59	2.49	0.96
111U	$6\Delta^3(1-\Delta)^3$	6.78	12.59	2.21	4.26	3.96	11.91	2.49	7.04
030T	$6\Delta^3(1-\Delta)^3$	6.78	6.99	2.21	1.32	3.96	1.03	2.49	1.01
030C	$2\Delta^3(1-\Delta)^3$	2.26	0.00	0.74	0.15	1.32	0.00	0.83	0.00
201	$3\Delta^4(1-\Delta)^2$	6.58	2.80	4.69	5.88	0.66	4.26	0.31	2.44
120D	$3\Delta^4(1-\Delta)^2$	6.58	2.80	4.69	4.85	0.66	0.00	0.31	0.07
120U	$3\Delta^4(1-\Delta)^2$	6.58	23.08	4.69	8.09	0.66	2.06	0.31	1.50
120C	$6\Delta^4(1-\Delta)^2$	13.16	0.35	9.38	2.94	1.32	0.15	0.63	0.07
210	$6\Delta^5(1-\Delta)$	25.55	17.48	39.71	33.53	0.44	1.18	0.16	0.86
300	Δ^6	8.27	17.48	28.03	33.38	0.02	1.18	0.01	0.71

Triad Census

Limitations of triad census:

Triads are **not independent** of one another

- A change in a single arc affects the isomorphism classes of N-2 triplets

Quantification of the observed isomorphism classes is **deterministic**

- What if the observed triad distribution is one instantiation of a probabilistic tie generating mechanism?

More sophisticated **parametric statistical models** (e.g., Exponential Random Graph Models) are available

- Controls for size, density, dyad types, and other higher order subgraphs
- These models enable asking questions such as, “controlling for the number of cycles in this network, how much effect does transitivity have on tie formation?

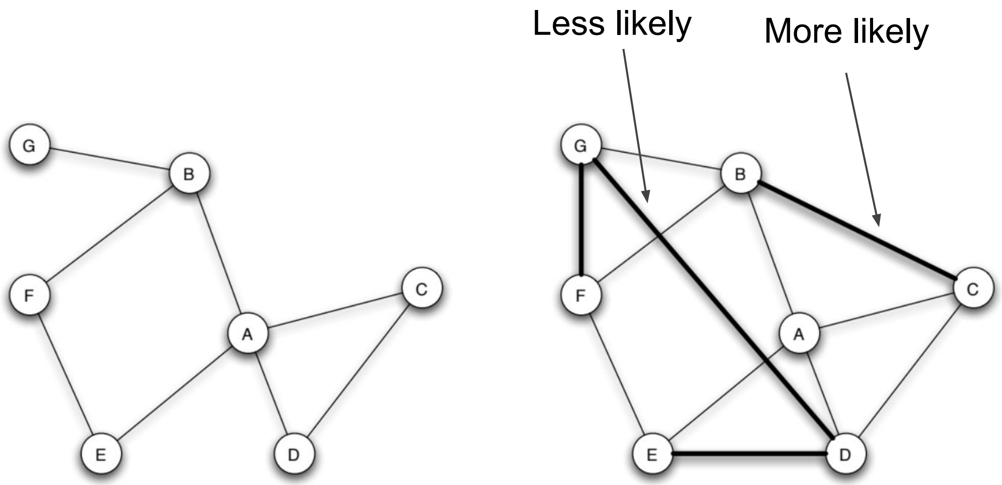
Triadic Closure and Clustering

Triadic Closure

A social network is constantly evolving

- New ties form
- Existing ties decay

In this process, **two nodes that are connected to the same set of other nodes have a higher probability of forming an edge**



(a) *Before new edges form.*

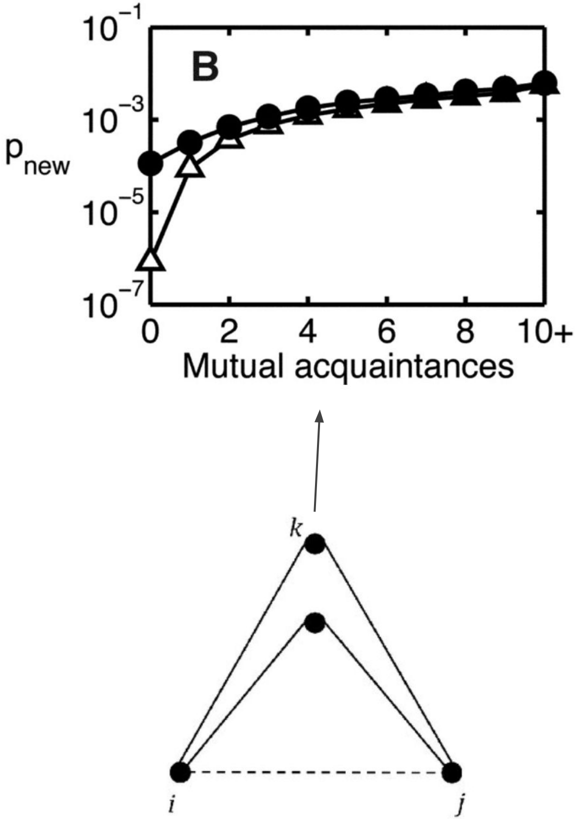
(b) *After new edges form.*

This results in a high frequency of fully connected triads in a network.

Triadic Closure

The more friends in common, the more likely a new tie forms

Email network of students, faculty, and staff at a university



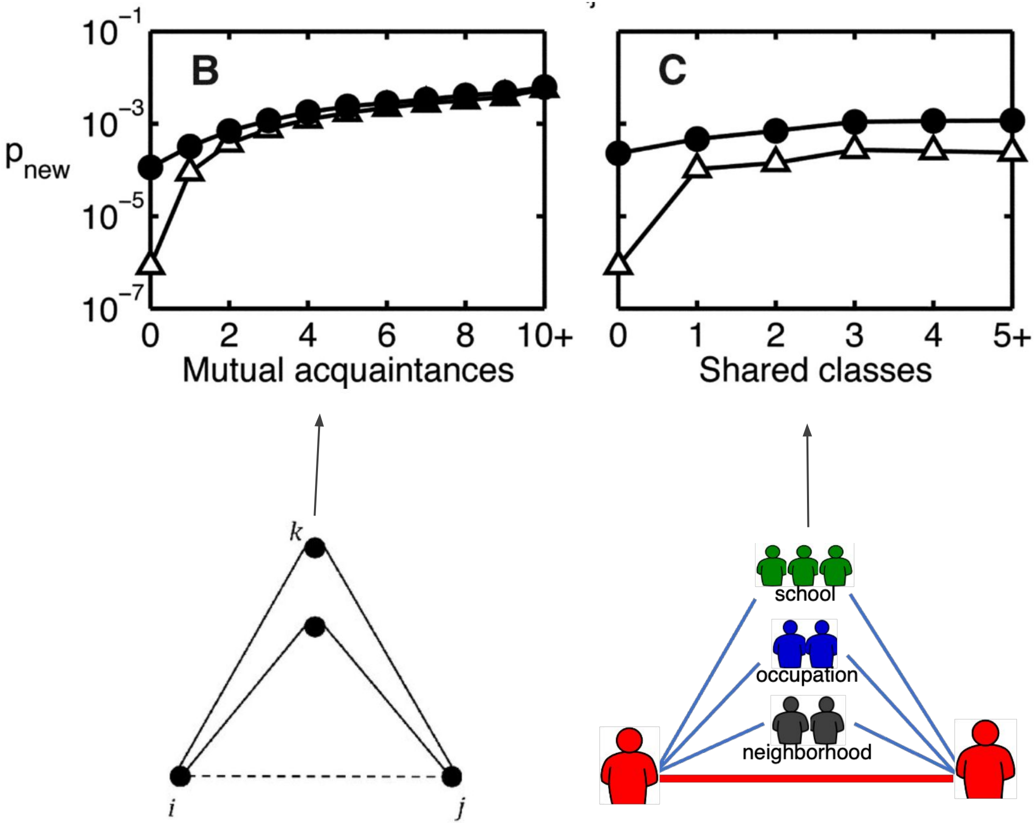
Kossinets and Watts 2006

Triadic Closure

The more friends in common, the more likely a new tie forms

This effect is stronger when the dyad also shares similar social contexts (e.g., two students overlap in multiple classes)

Email network of students, faculty, and staff at a university



Kossinets and Watts 2006

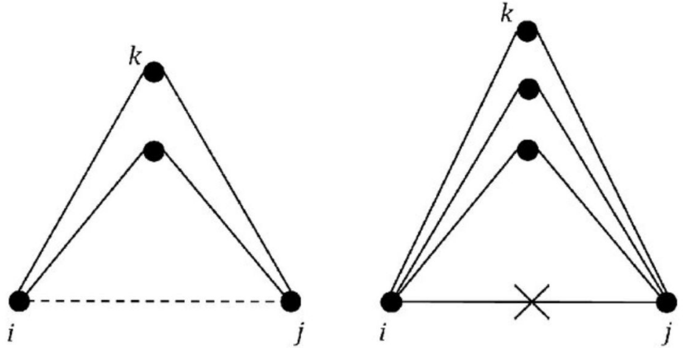
Triadic Closure

Furthermore, an *i-j* tie that closes many triads:

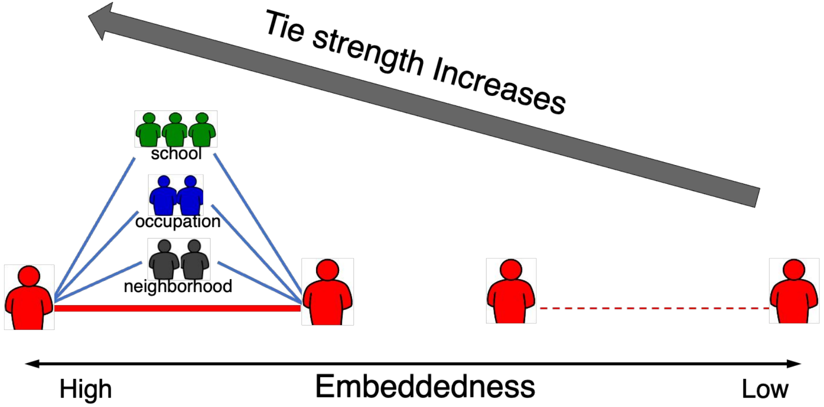
- is less likely to dissolve
- tends to be **stronger** (higher interaction frequency, positive emotions, reciprocity)

In networks parlor, the *i-j* tie is highly **embedded** in common network neighbors

→ Recall the stabilizing force of triads that Simmel argued



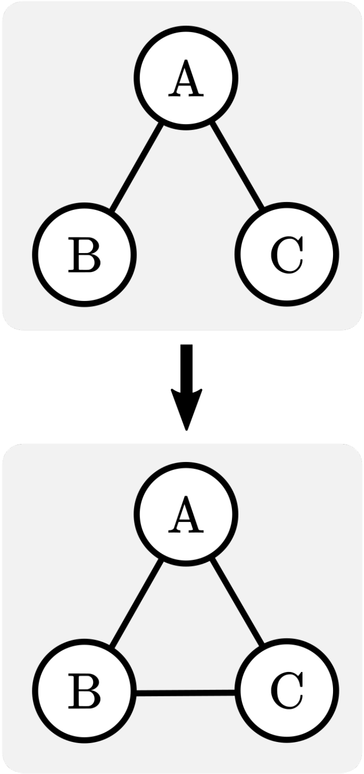
Twitter mention network (left) and phone call network (right)



Triadic Closure

Why do social networks exhibit strong triadic closure?

- **Opportunity**: If B and C each often spend time with A, they are likely to meet each other
- **Trust**: B and C can trust each other because they trust A
- **Incentives**: A introduces B to C because A believes more can be achieved if the three collaborate
- **Cognitive dissonance**: If A is good buddies with B and C, A may be distressed by the cognitive dissonance → Restore structural balance by introducing the two
- **Homophily**: The three all share a common interest, so A is friends with B and C. There is a good chance B and C meet each other even without A's introduction (What a small world!)

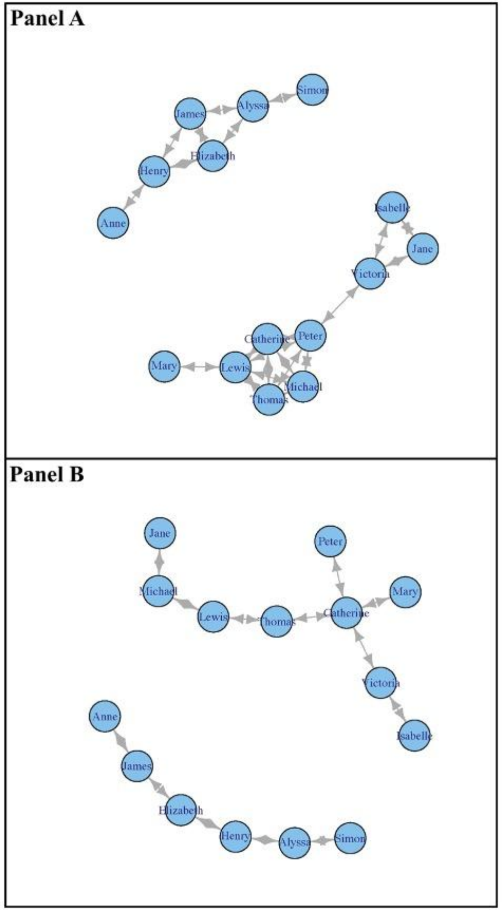


Triadic Closure

Our brain may have evolved to perceive triadic closure

People more accurately remember who is connected to whom in a network with more triadic closure

→ Triadic closure is a sort of information compression heuristic, which may have given an evolutionary advantage



Brashears and Quintain 2015

Measurement of Triadic Closure

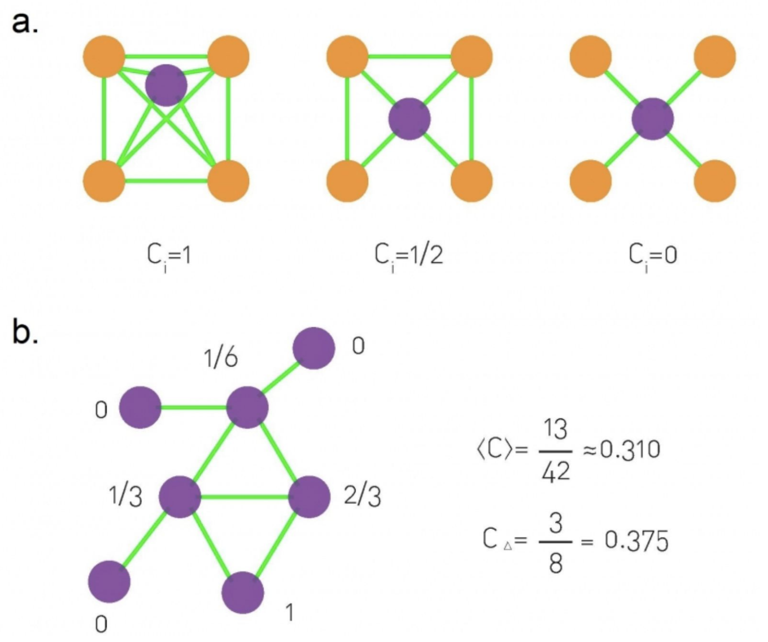
The extent of triadic closure in a network:

- Local clustering coefficient: The probability that two neighbors of a node are connected

$$C_i = \frac{2L_i}{k_i(k_i - 1)}$$

Number of ties among *i*'s neighbors (excluding ties involving *i*)

The average across *all* nodes is that network's "local" clustering coefficient

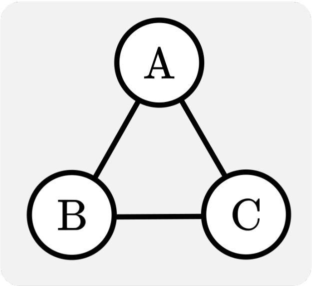


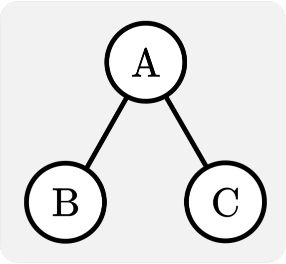
Measurement of Triadic Closure

The extent of triadic closure in a network:

- Global clustering coefficient (i.e., transitivity)

$$C_{\Delta} = \frac{3 \times Number\ Of\ Triangles}{Number\ Of\ Connected\ Triples}$$





This is the ratio of the number of closed triads to the number of open triads in a network

Summary

A unique feature of social networks is the dynamics of triads

- Triad is the most elementary group
- Structural balance
- Transitive and cyclical triads
- Triad census
- Triadic closure
- Measures of clustering

Edge vs. Social Tie

Social Ties are "Messy"

Edge vs. Social Tie

An edge in a graph is devoid of “meaning” or “content”

Its very utility comes from abstraction

Edge vs. Social Tie

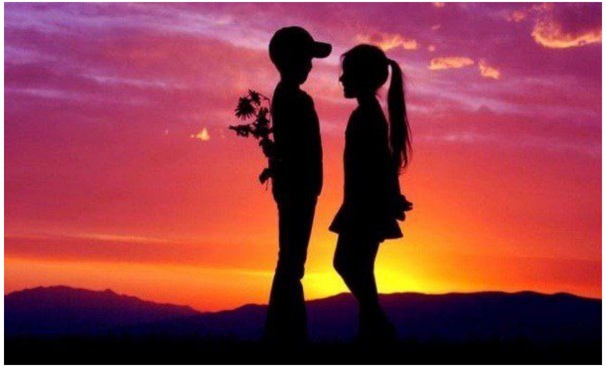
An edge in a graph is devoid of “meaning” or “content”

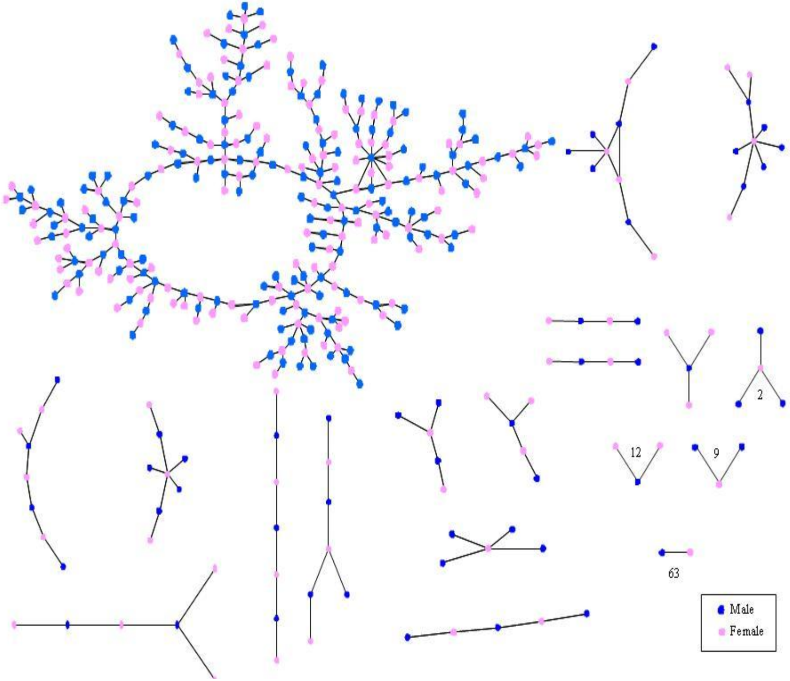
Its very utility comes from abstraction

A social tie in a network is a hefty baggage: carries emotion, meaning, norms, expectations, trust, social roles, and a history

Edge vs. Social Tie

Yet, the messy, variegated content in that baggage can surprisingly constrain or enable the emergence of certain graph structures

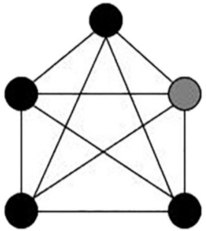




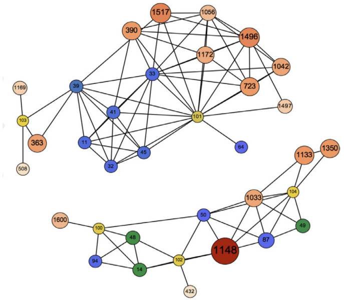
Edge vs. Social Tie

The inverse is also true: a graph structure can also constrain or enable certain characteristics in social ties

closed structure



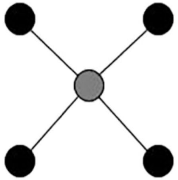
Monitoring, loyalty



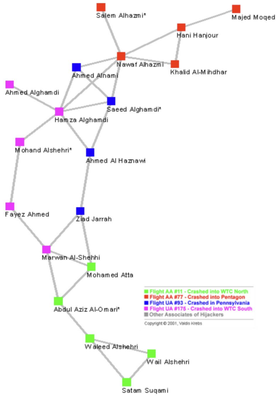
Mafia network

Agreste et al. 2016

open structure



Secrecy, discretion



Terrorist network

Krebs 2002

Social Ties and Social Support

People activate particular social ties for particular resources or support

Strong vs. weak ties

- Strong ties generally provide wide range of support

Physical contacts (e.g., neighbors)

- Provide small/large services (e.g., borrowing sugar, giving a ride to the station)
- Limited emotional and financial support

Kinship ties (e.g., parents, siblings)

- Emotional and/or financial support

Social Ties and Information Diffusion

Similar to social support, people selectively talk about certain topics to certain types of relationships

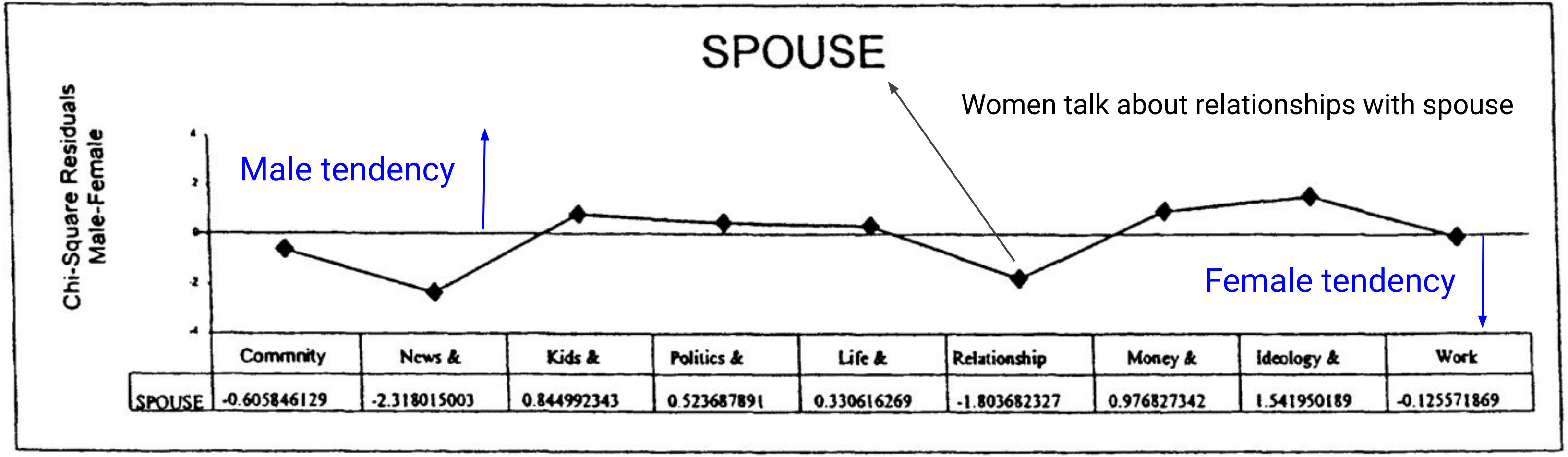
Examples:

- Sensitive topics (politics and religion) are usually discussed with close friends and family
- Generally, people discuss important matters with people they trust (i.e., confidants)
- Confidants potentially wield substantial influence on one's opinion
- At a more macro scale, Studying opinion dynamics with confidant networks rather than an all-encompassing network might yield more insight

Social Ties and Information Diffusion

So, with **whom** do we discuss important matters? And **what** are those important matters?

Panel a: Conversation Asymmetries for Talking with *Spouse*



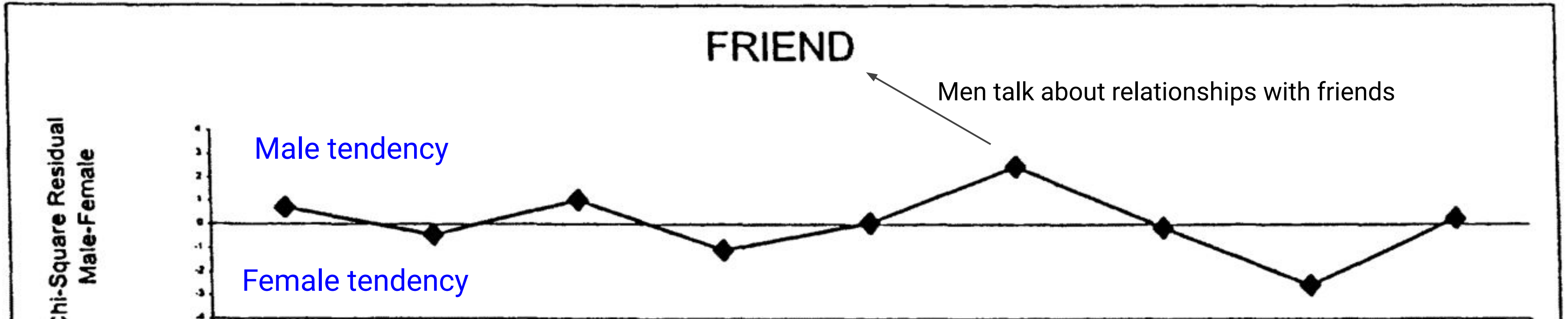
	Commnity	News &	Kids &	Politics &	Life &	Relationship	Money &	Ideology &	Work
SPOUSE	-0.605846129	-2.318015003	0.844992343	0.523687891	0.330616269	-1.803682327	0.976827342	1.541950189	-0.125571869

Bearman and Parigi 2004

Social Ties and Information Diffusion

So, with **whom** do we discuss important matters? And **what** are those important matters?

Panel b: Conversation Asymmetries for Talking with *Friend*



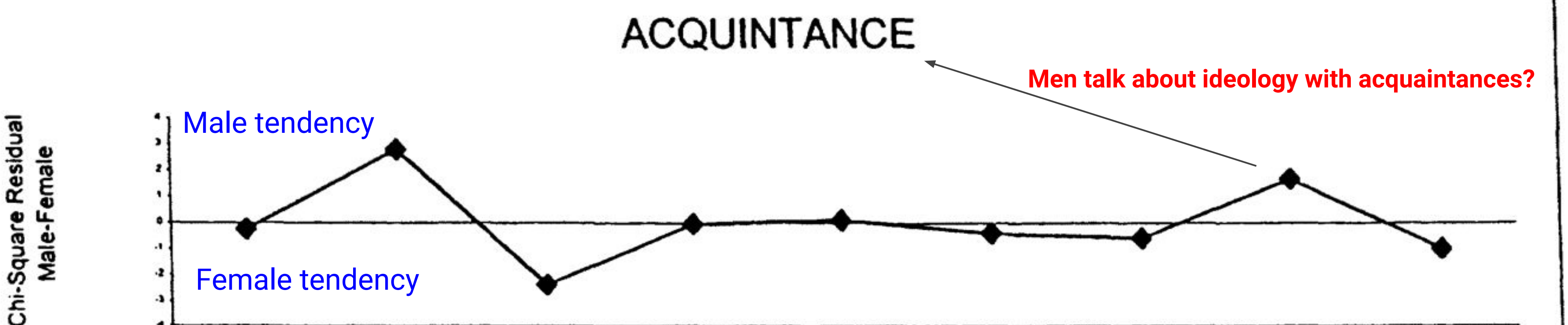
	Commnity Issue	News &	Kids &	Politics &	Life & Health	Relationships	Money & House	Ideology &	Work
SPOUSE	0.709342279	-0.448168813	1.024631883	-1.077416856	0.060591838	2.463100899	-0.131488919	-2.575398042	0.254459412

Bearman and Parigi 2004

Social Ties and Information Diffusion

So, with **whom** do we discuss important matters? And **what** are those important matters?

Panel d: Conversation Asymmetries for Talking with *Acquaintance*



	Commnity Issue	News &	Kids &	Politics &	Life & Health	Relationships	Money & House	Ideology &	Work
SPOUSE	-0.25857459	2.796870511	-2.39382851	-0.045949224	0.120480897	-0.404151437	-0.596462518	1.664829837	-0.989515895

Bearman and Parigi 2004

Social Ties and Information Diffusion

Men talk about ideology with acquaintances…

But, don’t people discuss important, often private, topics with their trusted strong ties?

Answer: Not necessarily. “People may often confide in people they do not even consider confidants (Small 2017).”

Social Ties and Information Diffusion

Why?

Strong ties (e.g., friends and family) are interconnected (i.e., triadic closure)

- Disclosing sensitive/embarrassing information to one friend can quickly spread to other close friends in the same social circle

Social Ties and Information Diffusion

Why?

Strong ties (e.g., friends and family) are interconnected (i.e., triadic closure)

- Disclosing sensitive/embarrassing information to one friend can quickly spread to other close friends in the same social circle

Topic-alter dependency

- Strangers share very few social contexts, so people feel safe to disclose some sensitive topics

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The Surprising Boost You Get From Strangers

Sometimes a stranger—not a friend or a loved one—can significantly improve our day, providing comfort or helping to broaden our perspective

By *Elizabeth Bernstein* Follow

May 11, 2019 at 5:30 am ET

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Social Ties and Information Diffusion

So, will word about the exquisite cake from La Gourmandine spread like wildfire at the party?

Not necessarily, because **valued information** about scarce goods might spread through highly exclusive **strong ties**

Alternatively, gourmet cake as a topic might be discussed between cake lovers, but not with, say, people who are sensitive to gluten/dairy or who are indifferent → **topic-alter dependency**

Social Ties and Information Diffusion

Network science provides powerful tools for modeling social phenomena

Yet, if the ties that are used to construct the network inadequate for the phenomenon under study, the analysis and its conclusions will be irrelevant

Hence, qualitative aspects of social ties (the “messy” content) must be carefully evaluated

- Meticulously consider what social ties for constructing the network makes sense, given the goals of the analysis

Summary

An interpersonal tie influences and is influenced by the broader network structure

- Social support differs by type of relationship
- Topic-alter dependency can affect information diffusion
- Social tie can create a graph signature
- Dynamics of social ties hold implications for network structure