Network Analysis:

The Hidden Structures behind the Webs We Weave

17-213 / 17-668

Power and Centrality in Social Exchange

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2-min Quiz, on Canvas

Quick Recap – Last Thursday’s Lecture

Power, influence, prominence of an individual is relational: They originate from relationships that the individual has with others

Network centrality quantifies this relational view of power

- Degree centrality
- Closeness centrality
- Betweenness centrality
- Eigenvector centrality

Centrality measures tend to be correlated (e.g., degree and closeness), but they quantify different facets / dimensions of power

Where Centrality Breaks: Positive Sum vs. Zero-Sum Relations

The Social Exchange Perspective

All human interactions are "exchanges" that are social by nature

- Market exchange of goods/services
- Non-market exchange: gift, favors, advice, respect, emotion, invitation, etc.
- Interdependence (needs/wants) drives social interaction in the form of exchange

Trust is the basis of all exchanges (and social interactions)

- "How can I be sure that they won't cheat?"
- Market vs. non-market exchanges use different mechanisms to solve the problem of trust
- **What** are they?

The Social Exchange Perspective

Market exchange:

- Terms of exchange: Negotiation
- Time frame: Immediate (spot market)
- Enforcement: Institutional sanctions, formal law
- Problem of trust: solved by central enforcers (state)
- Zero-sum: A’s profit is B’s cost

Non-market exchange:

- Terms of exchange: Reciprocity
- Time frame: Unspecified
- Enforcement: Social pressure, norms
- Problem of trust: solved by the decentralized collective (reputation, ostracism)
- Not clearly zero-sum: A’s profit does not directly mean B’s cost

The Social Exchange Perspective

Social ties we use to construct networks and the network measures we apply implicitly assume social interactions to resemble non-market exchange

- Reciprocity
- Social pressure discourages norm violation (trust from triadic closure)
- Power and influence grows with having more exchange partners (centrality)
- Not clearly zero-sum: A’s social support to B can be reciprocated at a later time in-kind or with different resources (e.g., labor, status, loyalty)

Positive vs. Negative Connections

Network measures cannot be blindly applied to any network

Example: The Interdependence between ties
one exchange relation is contingent on the (non)exchange in a neighboring relation

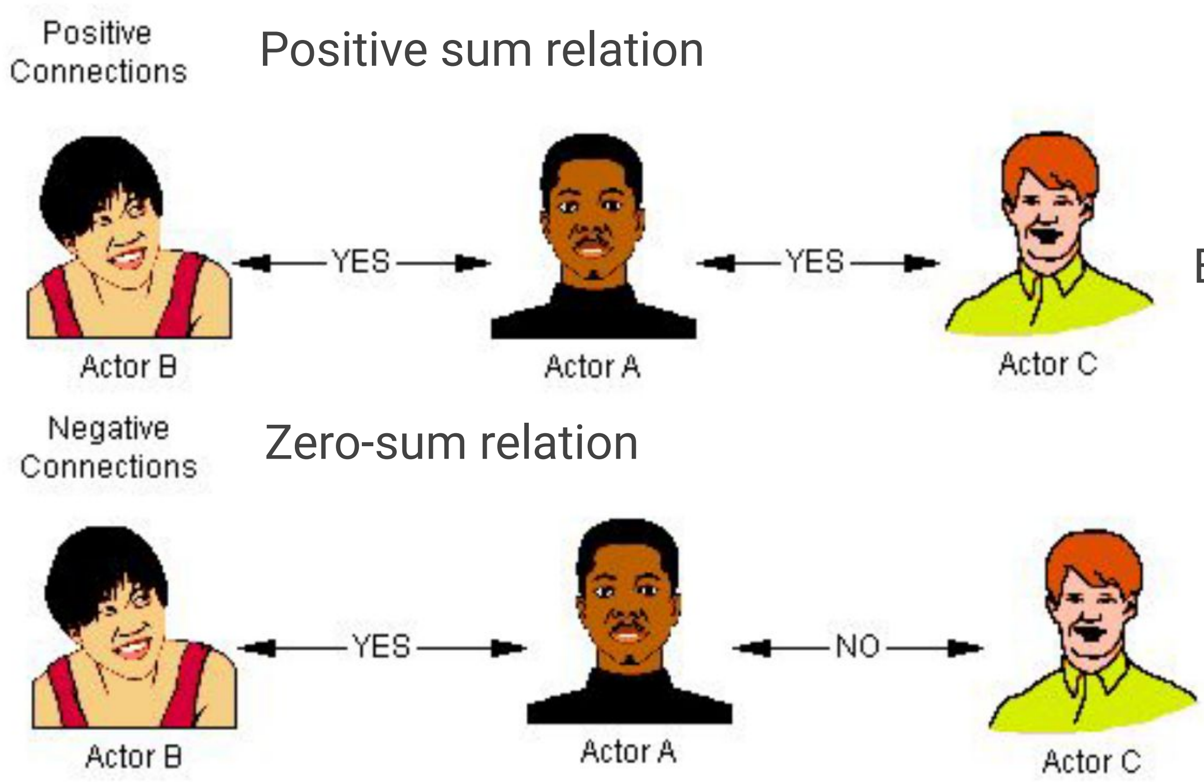
- positive connections: Flow of resources from B → A → C. C can receive resources from A only if B transfers them to A.

- negative connections: zero-sum relations. A's exchange with B implies that A does not need to exchange with C → B's gain is C's loss

Positive vs. Negative Connections

Positive sum relation

Zero-sum relation



Example: supply chain

Example: One buyer and two competing sellers

Positive vs. Negative Connections

Degree centrality predicts “power” in networks of positive connections

Question: Does centrality predict power in networks of negative connections (i.e., zero-sum relations)? **Why**?

Power-Dependence Theory

For negative connections:

- If A depends on B more than B depends on A → B has that much more power over A (Emerson, 1962)

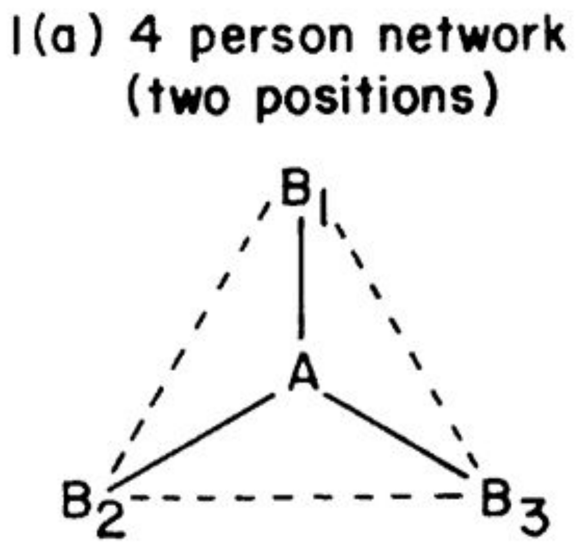
$$P_{AB} = D_{BA}$$

- These dependencies (hence power) stem from positions in the exchange network

So, does centrality and power-dependence logic make the same predictions about powerful positions in negatively connected networks?

Centrality in Negatively Connected Exchange Networks

Exchange network experiment by Cook et al. (1983)



Lines: Exchange opportunities

- **Solid lines**: More profitable (24 points total)
- **Dashed lines**: Less profitable (8 points total)
-

Alphabets: Exchange positions

- Same alphabet positions (e.g., B1 and B2) are identical

Local knowledge: Participants do not know the exchange network structure (only the ties that they have)

Centrality in Negatively Connected Exchange Networks

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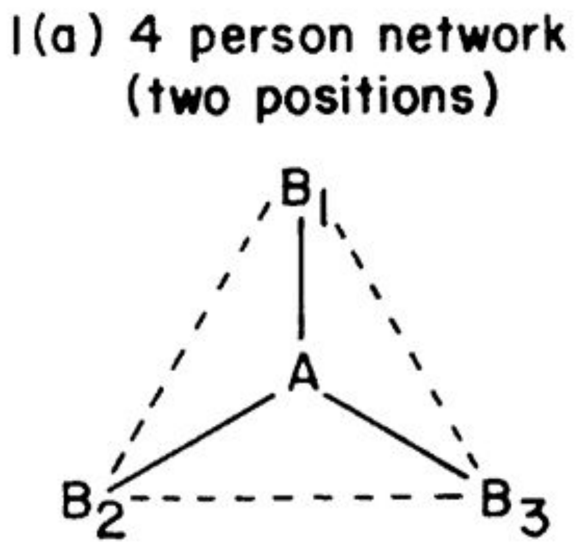


Network 1(a): A can exchange with only one among B1, B2, and B3 in one round

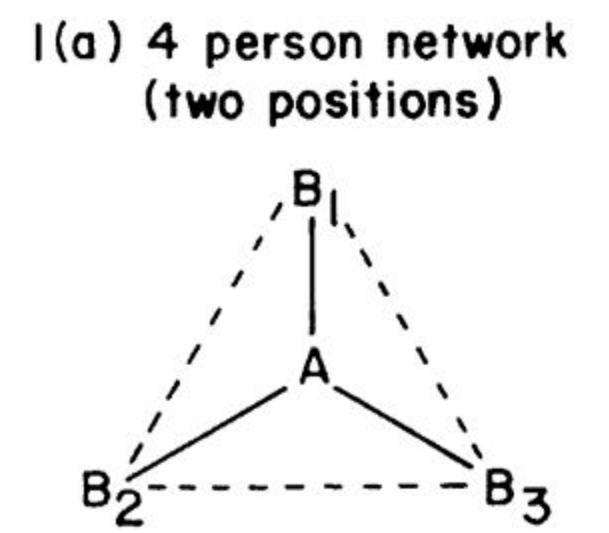
A and a partner in position B can negotiate how to split 24 points (solid line)

B1 and B2 can negotiate how to split 8 points (dashed line)

Negatively connected: If A chooses B1 as partner, then B2 and B3 cannot exchange with A

Predictions of Power According to Centrality

Exchange network experiment by Cook et al. (1983)



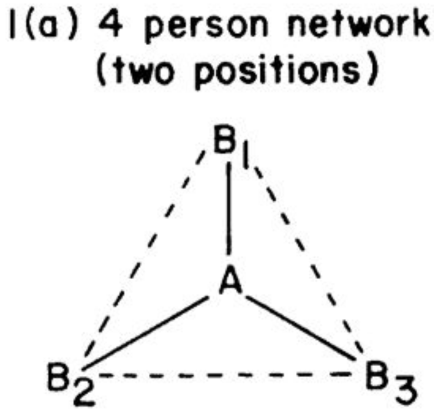
Power: Measured by the total points that an occupant of a position earns through multiple rounds of exchanges

Centrality: A is the most central in terms of **weighted** degree, closeness, and betweenness centrality

Therefore, centrality predicts that power should be
A > B1 = B2 = B3

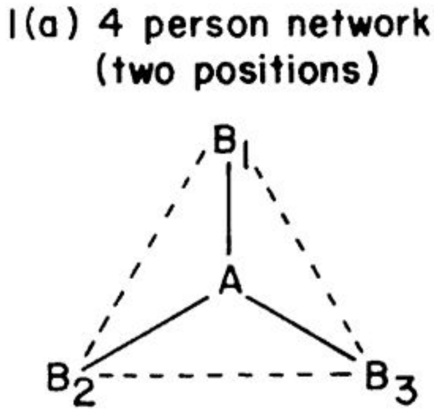
Predictions of Power According to Power-Dependence Theory

Power-dependence theory Prediction



If B1 exchanges with B2 or B3 → 4 points
If B1 exchanges with A → any point above 4 is more beneficial
If B1 suggests to A for an equal split (12 points each), A can refuse and negotiate with B2 or B3 for a better deal
What is the maximum that A will likely get?

Predictions of Power According to Power-Dependence Theory



At equilibrium: A's expected payoff: 20, B's expected payoff: 4

B position is dependent on A position to maximize payoff
→A's power over B is equal to B's dependence on A

Power: A > B1=B2=B3

(Same as centrality)

Predictions of Power According to Centrality

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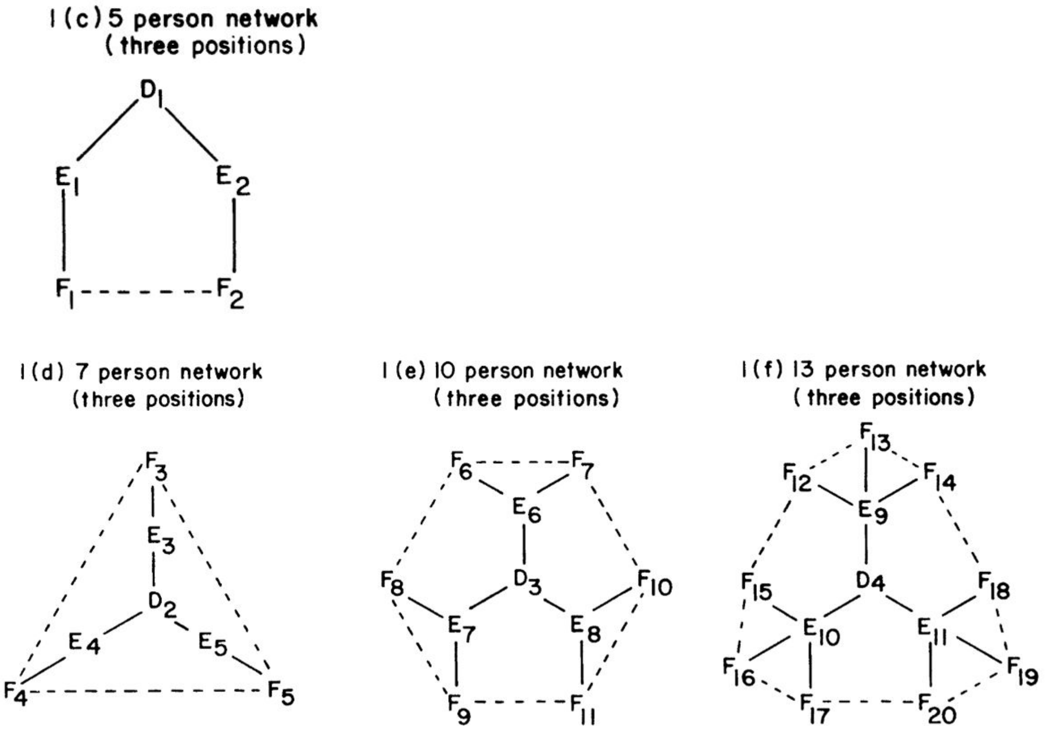


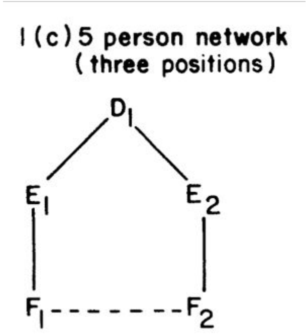
D is the most central in terms of (weighted) **closeness and betweenness** for all configurations
Power: D > E > F

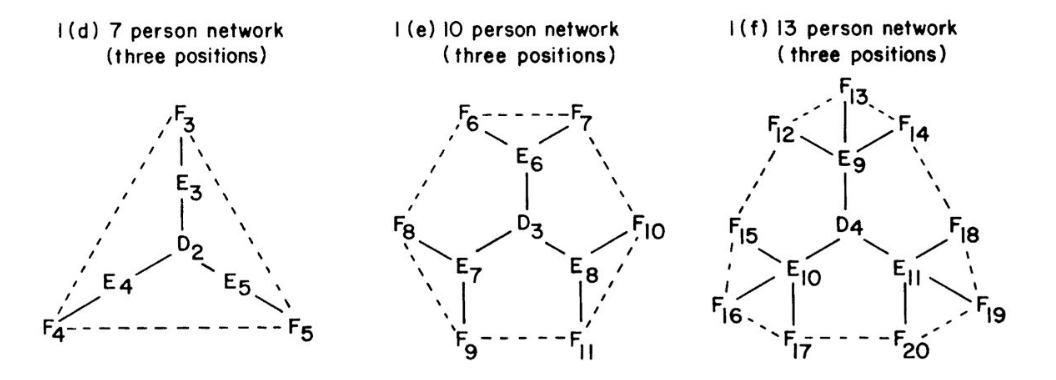
D and **E** have same **degree** centrality in 1c and 1e
Power: D = E > F

D has larger degree than **E** in 1d
Power: D > E > F

Power: E > D > F for 1f

Predictions of Power According to Power-Dependence Theory





Who is the most powerful according to power-dependence theory?

F is dependent on E for higher payoff
→ E can ask for 20 points to F
→ E can also ask D for a “price match” (20 points)

D cannot easily earn more than 4 points because all Es can turn to the Fs for that much

D’s and F’s expected payoffs will be 4

Power: E > D = F

Experimental Evidence

Experiment designed with network 1c

Recruited 100 university students

27 transaction rounds

Negotiate with connected partners each round

Only one transaction per round per person

- negatively connected

Transactions are not revealed to others

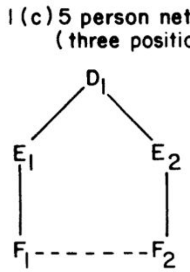


TABLE 1

EXPERIMENTAL RESULTS:

MEAN PROFIT OF PERSON E PER EXCHANGE WITH D AND WITH F in NETWORK 1*c* BY EXCHANGE INCENTIVE AND TRIAL BLOCK

EXCHANGE INCENTIVE AND EXCHANGE PARTNER	TRIAL BLOCKS 1	2	3
Low:			
D	13.80	12.69	13.32
	(4.13)	(4.26)	(4.25)
F.	13.27	14.78*	15.44**
	(3.10)	(2.77)	(2.96)
High:			
D	12.90	13.72	17.19**
	(3.71)	(4.40)	(5.26)
F.	15.52**	16.66**	16.91**
	(2.38)	(2.10)	(2.46)
Combined:			
D	13.35	13.21	15.26**
	(3.95)	(4.36)	(5.16)
F.	14.40**	15.72**	16.18**
	(2.99)	(2.63)	(2.82)

NOTE —The profit obtained by D and F in negotiations with E can be obtained by subtracting the values in this table (E's profit) from 24 Standard deviations are in parentheses

* Significantly greater than 12 ($P <$ 05)

** Significantly greater than 12 ($P <$ 01)

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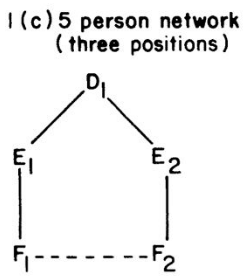

Why was E's power realized clearly in the last 9 rounds (Block 3)?

Why didn't E reach theoretical maximum (20 points)?

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Simulation Evidence

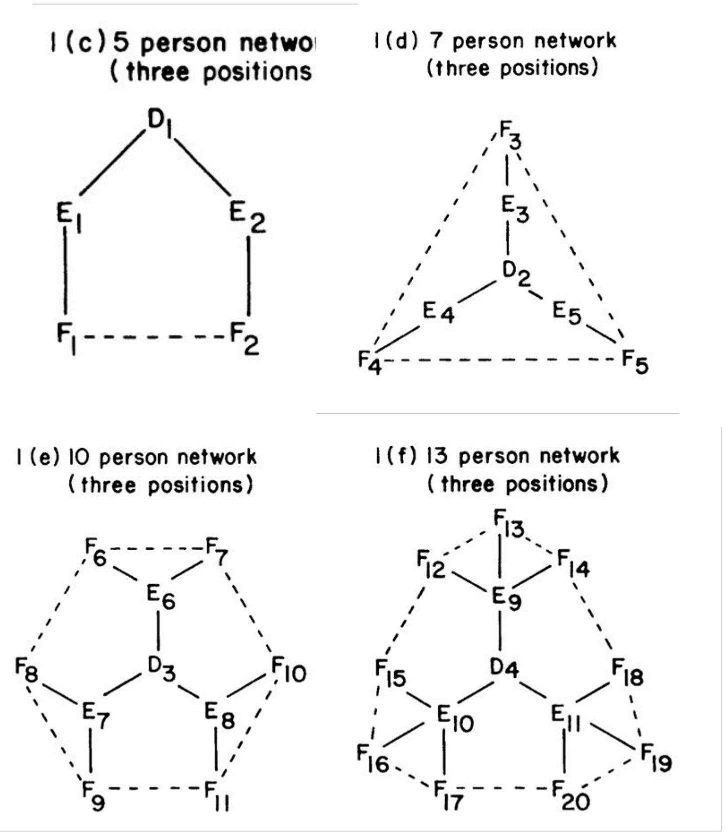


TABLE 2

SIMULATION RESULTS:

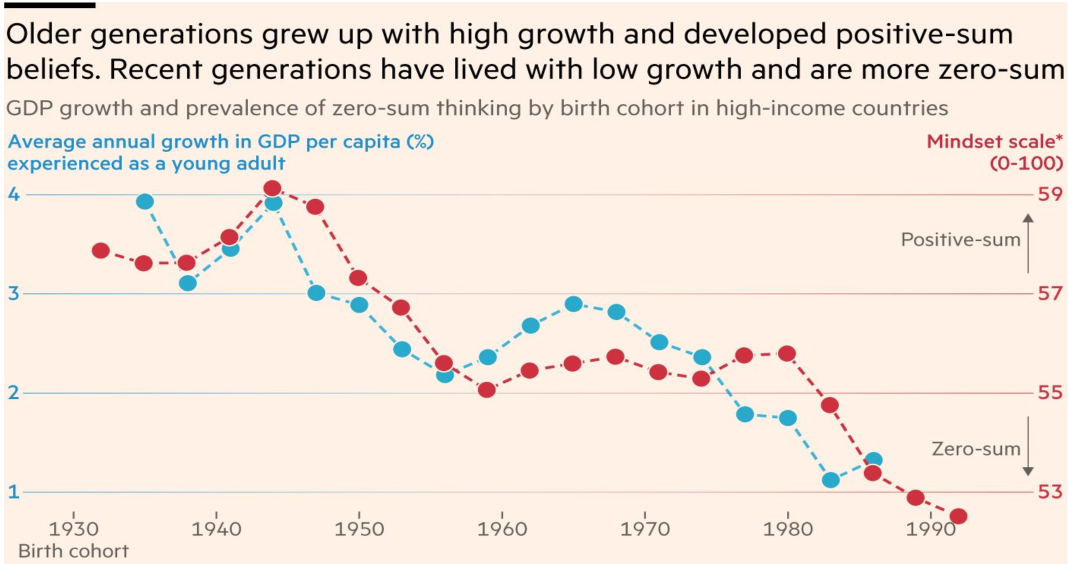
MEAN PROFIT OF THE POWERFUL (E) PER "EXCHANGE" WITH D AND WITH F IN FOUR REPLICATIONS VARYING NETWORK SIZE

FIGURE PART, SIZE OF NETWORK, AND E'S EXCHANGE PARTNER	TRIAL BLOCKS 1	2	3	4	5	6
1*c*, 5 actor:						
D	12.79a	15.36	17.15	18.60	19.31	19.55
	(2.57)	(2.76)	(2.98)	(2.11)	(1.67)	(1.44)
F............	14.71	16.33	17.83	19.08	19.86	19.91a
	(2.64)	(2.76)	(2.56)	(1.86)	(1.42)	(1.26)
1*d*, 7 actor:						
D	10.56b	13.03	15.00	16.54	17.64	18.47
	(3.02)	(3.07)	(2.81)	(2.32)	(1.68)	(1.43)
F............	14.33	15.31	16.64	17.79	18.66	19.06
	(2.95)	(2.34)	(2.40)	(1.93)	(1.38)	(.97)
1*e*, 10 actor:						
D	13.99	17.69	19.65	20.06	20.11	20.11
	(2.89)	(2.37)	(1.07)	(.58)	(.55)	(.54)
F............	16.35	18.68	19.86	20.11	20.22	20.01
	(2.16)	(1.35)	(.59)	(.42)	(.43)	(.39)
1*f*, 13 actor:						
D	14.50a	19.56c	20.42	20.63	20.50	20.43
	(3.18)	(2.10)	(.86)	(.66)	(.64)	(.55)
F............	17.18	20.06	20.67	20.87	20.69	20.58
	(1.69)	(.74)	(.57)	(.59)	(.52)	(.49)

NOTE.—These values represent the average profit E obtained in "exchanges" with D and F, with 24 units of profit available for each "exchange"; therefore D's and F's average profit equals 24 − E's profit in each case. Each trial block contained nine trials. Cell values are based on the simulation of 50 groups; in an occasional group, however, E did not complete an "exchange" in a given trial block. Cell means labeled "a" are based on 49 groups, that labeled "b" has 47 groups, that labeled "c" has 42 groups per cell; all others have 50 groups per cell. Standard deviations are in parentheses.

Zero-Sum Relations

Centrality does not accurately capture power in networks of zero-sum relations
Then, centrality might not predict power in a society where people believe social life is zero-sum



*100 = "Wealth can grow so there's enough for everyone"; 0 = "People can only get rich at the expense of others"
Sources: FT analysis of World Values Survey; Maddison Project database
Based on *Zero-Sum Thinking and the Roots of US Political Divides (Chinoy et al., 2023)*
FT graphic by John Burn-Murdoch / @jburnmurdoch

Power Centrality: A Synthesized Measure

Incorporating Negative Connections

Phillip Bonacich (inventor of eigenvector centrality)

- Proposes modification to eigenvector centrality (Bonacich 1987)

Insight: The source of power comes from

- Connections with powerful actors (positive connections)
- Connections with dependent actors (negative connections)
  - Those who do not have alternative options for exchange

Eigenvector centrality squarely captures power in positive connections

$$\lambda C_E(i) = \sum_j A_{ij}\, C_E(j)$$

A modified measure should make a node central to the extent that neighbors are less central

Bonacich Power Centrality

Beta parameter determines the importance of the centrality of the neighbors

Beta > 0: higher neighbor centrality increases my centrality
→ Connections with powerful actors

Beta < 0: higher neighbor centrality decreases my centrality
→ Connections with dependent actors

Beta = 0: Degree centrality

Eigenvector Centrality

$$\lambda C_E(i) = \sum_j A_{ij} \, C_E(j)$$

Power Centrality

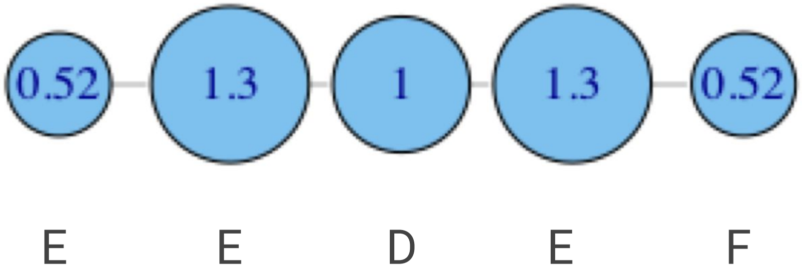
$$c_i(\beta) = \sum_j (\alpha + \beta c_j) A_{ji}$$

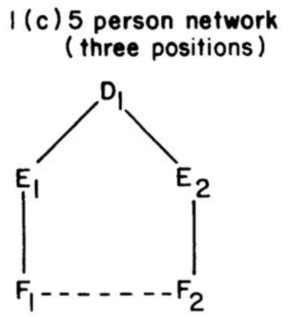
Bonacich Power Centrality

Example:

β=.25

β=-.25





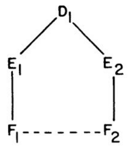
Bonacich Power Centrality

TABLE 3

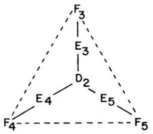
CENTRALITY SCORES FOR FOUR NETWORKS FOR SELECTED VALUES OF β

	NETWORK											
	1*c*			1*d*			1*e*			1*f*		
POSITION β	*D*	*E*	*F*	*D*	*E*	*F*	*D*	*E*	*F*	*D*	*E*	*F*
−.5	.00	1.58	.00	...	...	...	...	...	...	...	...	...
−.4	.73	1.45	.36	1.62	1.08	.54	−1.00	1.67	−.33	−1.72	1.53	−.57
−.3	.97	1.34	.49	1.62	1.08	.54	.36	1.81	.12	−.55	2.03	−.18
−.2	1.09	1.27	.54	1.62	1.08	.54	1.00	1.67	.33	.44	2.05	.15
−.1	1.15	1.23	.58	1.62	1.08	.54	1.30	1.55	.43	1.01	1.91	.34
0	1.20	1.20	.60	1.62	1.08	.54	1.46	1.46	.49	1.33	1.78	.44
.1	1.22	1.17	.61	1.62	1.08	.54	1.57	1.40	.52	1.52	1.67	.51
.2	1.25	1.16	.62	1.62	1.08	.54	1.63	1.36	.54	1.65	1.59	.55
.3	1.26	1.14	.63	1.62	1.08	.54	1.68	1.33	.56	1.74	1.53	.58
.4	1.27	1.13	.64	1.62	1.08	.54	1.72	1.30	.57	1.80	1.48	.60
.5	1.28	1.12	.64	...	...	...	...	...	...	...	...	...

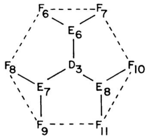
1(c) 5 person network (three positions)



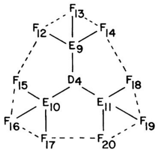
1(d) 7 person network (three positions)



1(e) 10 person network (three positions)



1(f) 13 person network (three positions)



Robustness vs. Fragility

Creative Variations on Power Centrality

Bothner et al. applies the recursive intuition in Bonacich's power centrality

One's structural fragility is a function of the fragility of the alters

Insight:

- Fragility roughly means too much reliance/dependence on few people
- The position is even more fragile if those few people are also in fragile positions

Structural Fragility

Herfindahl Index (*H*): Measures concentration

$$H_i = \sum_{j=1}^{n-1} d_{ij}$$

$$d_{ij} = \left[\frac{x_{ij}}{\sum_{j=1}^{n-1} x_{ij}}\right]^2$$

Proportion squared

x_{ij} Tie weight of *i*-*j* edge

The more that i's weight is concentrated to fewer alters, i's fate is greatly affected by those few

Structural Fragility

Use the herfindahl index matrix instead of the adjacency matrix

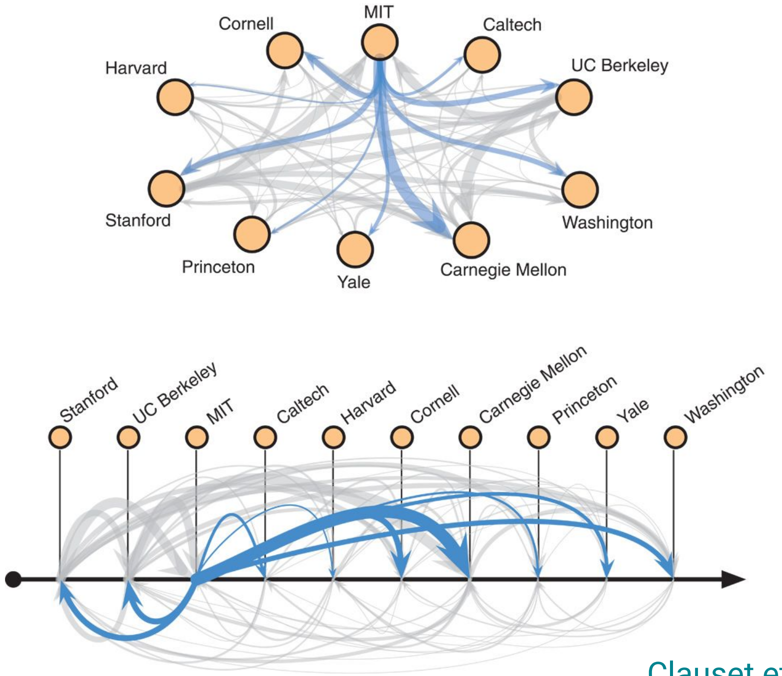
$$d_{ij} = \left[\frac{x_{ij}}{\sum_{j=1}^{n-1} x_{ij}}\right]^2$$

$$F_i(a, b) = \sum_j (a + bF_j)\, d_{ij}$$

Creative Variations on Power Centrality

Example: University department prestige and fragility in the network of faculty hiring

PhD faculty job placement network is hierarchical



Clauset et al. 2015

Fig. 1 Prestige hierarchies in faculty hiring networks.

(**Top**) Placements for 267 computer science faculty among 10 universities, with placements from one particular university highlighted. Each arc (u,v) has a width proportional to the number of current faculty at university v who received their doctorate at university u ($\neq v$). (**Bottom**) Prestige hierarchy on these institutions that minimizes the total weight of "upward" arcs, that is, arcs where v is more highly ranked than u.

Creative Variations on Power Centrality

Example: University department prestige and fragility in the network of faculty hiring

“A fragilely located department is one that trades scholars [faculty hiring between department *i* and *j*] with a limited set of departments that are similarly restricted in their set of exchange partners (Bothner et al. 2010).”

Creative Variations on Power Centrality

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"A fragilely located department is one that trades scholars [faculty hiring between department *i* and *j*] with a limited set of departments that are similarly restricted in their set of exchange partners (Bothner et al. 2010)."

TABLE 7
MODELS PREDICTING DEPARTMENTAL PRESTIGE IN BURRIS'S PHD EXCHANGE NETWORK

	1	2	3	4
Social capital	1.118	1.105	.747	1.068
	(.069)**	(.067)**	(.045)**	(.078)**
Fragility ($c = 0$)				−.085
				(.093)
Fragility ($c = .99$)		−.101	−.093	−.096
		(.039)*	(.036)*	(.039)*
Article publications	.072	.067	.070	.064
	(.051)	(.049)	(.052)	(.049)
Citations	.005	.005	.026	.005
	(.010)	(.010)	(.053)	(.010)
Research grants	−.000	−.001	−.008	−.000
	(.003)	(.003)	(.045)	(.003)
Weighted article publications	.180	.176	.112	.165
	(.082)*	(.079)*	(.051)*	(.080)*
Book publications	.245	.217	.114	.232
	(.090)**	(.088)*	(.046)*	(.089)*
Constant	−.401	−.294	−.000	−.138
	(.150)**	(.151)	(.034)	(.227)
N	94	94	94	94
R^2	.89	.90	.90	.90

Creative Variations on Power Centrality

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Question: What do c=0 and c=0.99 mean?

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N	94	94	94	94
R^2	.89	.90	.90	.90

Summary

Different centrality measures for different aspects of power

Ask if centrality is the right way to think about power, given the nature of the tie (positive vs. negative connections)

Centrality does not quantify power accurately in negatively connected exchange networks

Power-dependence theory gives better prediction

Bonacich power centrality modifies eigenvector centrality to measure centrality in negative connections

Creative variation: Structural fragility